Monthly Liquidity-adjusted Value-At-Risk Using Daily Returns

Maxime Derbier

A Thesis in

The John Molson School of Business

Department of Administration

Presented in Partial fulfillment of the Requirements

of the Degree of Master of Science in Administration (Finance) at

Concordia University

John Molson School of Business

Montreal, Quebec, Canada

March 2014

CONCORDIA UNIVERSITY

School of Graduate Studies

This is to certify that the thesis prepared

By: Maxime Derbier

Entitled: Monthly Liquidity-adjusted Value-At-Risk Using Daily Returns

and submitted in partial fulfillment of the requirements for the degree of

Master of Science in Administration (Finance)

complies with the regulations of the University and meets the accepted standards with respect to originality and quality.

Signed by the final Examining Committee:

______________________________ Dr. Xiao Huang

______________________________ Dr. Lawrence Kryzanowski

______________________________ Dr. Rahul Ravi

______________________________ Dr. Ravi Mateti

Approved by ____

Chair of Department or Graduate Program Director

_____________ 2014 ______________________________

Dean of Faculty

Monthly Liquidity-adjusted Value-At-Risk Using Daily Returns

ABSTRACT

Monthly Liquidity-adjusted Value-At-Risk Using Daily Returns

Maxime Derbier

The purpose of this thesis is to apply liquidity proxies from the microstructure literature based on daily data to estimate the liquidity cost within a monthly parametric Value-at-Risk (VaR) model to create a parsimonious yet relevant measure of both market and liquidity risk. The proxies examined in this thesis include the 'Roll Proxy' of Roll (1984), Zeros by Lesmond, Ogden and Trzcinka (1999), the Holden, Fong and Trzcinka (2011, FHT henceforth) proxy and the Shane and Schultz (2012) High-Low Spread proxy (HLS henceforth) as they are computationally less onerous, do not necessitate intraday data and were proven to be reliable in the literature. We find that the FHT, Roll and HLS proxies are indeed good estimators of effective costs making them appropriate additions to a VaR model, while the Zero proxy performs very poorly due to its overestimation of effective costs. With high correlations and similar distributional properties relative to the Hasbrouck effective costs, the FHT proxy performs best when back-tested against liquidity-adjusted returns relative to all other proxies.

Table of Contents

List of Tables and Figures

Monthly Liquidity-Adjusted Value-At-Risk Using Daily Returns

1. INTRODUCTION

As the last couple of years have shown, liquidity can be an important factor when calculating risk and can result in significant costs when unwinding a security position particularly in times of crises. Many of the recent financial meltdowns are due to liquidity shortages such as the 1987 stock market crash, the 1997 run on the Thai Bath, two large hedge fund breakdowns of LTCM in 1998 and Amaranth Advisors in 2006. During such periods, asset values became increasingly volatile due to the inability to liquidate large positions without significantly affecting prices adversely (Brunnermeier *et al.*, 2009; Bhyat, 2010). This was also an issue in the more recent subprime crisis of 2007-2008 where banks around the world were forced to sell off positions due to margin calls during a liquidity crunch which caused stock prices to plummet substantially across markets (Brunnermeier *et al.*, 2009).

Indeed, we can recognize self-perpetuating cycles between market crises and illiquidity where withdrawals of market participants from the liquidity pool can precipitate a market collapse (Bhyat, 2010; Naes *et al.*, 2011). Today's most popular tool to measure, control and manage financial risk is the Value-at-Risk (VaR henceforth) model inspired by JP Morgan's Riskmetrics. This model measures the worst expected loss over a given horizon at a certain confidence level. VaR models are subject to much scrutiny recently for not capturing outlier events (Campbell, 2010; Lazaregue-Bazard,

2010) and for only measuring possible loss under “normal” conditions without considering market liquidity (Contreras, 2010).

Thus, a VaR model is only as good as the quality of its parameters such as its estimation period, distributional assumptions, and volatility estimates. Some of the ad hoc methods employed by risk managers to account for liquidity are to artificially increase the volatility of positions or to lengthen the time horizon used in calculating VaR. For example, financial institutions are required by the Basel Accord to report their 10 day VaR to account for such periods of illiquidity. However, such methods are arbitrary and are conceptually correct only if the optimal liquidation periods for different assets corresponds to their actual holding periods.

A tenuous VaR assumption is that liquidity costs can be neglected as long as the liquidation horizon is long enough. Thus, there is no adjustment and positions are expected to be liquidated at mid-spread which assumes no friction in obtaining fair value. However, traders rarely realize mid-spreads when liquidating a position quickly and instead incur a cost from the bid-ask spread as well as from the potential price impact of a sale. Under the assumption that a position is transacted as a market order against available limit orders, the difference between the realized price and the mid-spread measures the price impact of the trade due to illiquidity. For small volumes, this is the bid-ask spread but for larger orders the price impact will be larger as liquidity is taken from prices at higher or lower levels away from the inside spread. As such, during periods of market turbulence, “normal” conditions no longer apply and the cost to liquidate a position may change drastically depending on the financial product in question.

While some more “risk off” products such as U.S. Government treasuries may fare better during periods of uncertainty due to a flight to quality, other more “risk on” products such as small cap stocks and emerging currencies may become very illiquid as investors try to lower their overall risk exposures. This not only does this increase the spread but it makes these latter products more prone to larger price fluctuations as fewer participants are present to provide liquidity when needed most. Therefore, in order to have a precise estimation of market risk, one must model the distribution of the deviations of the liquidation values from the mid-spread in order to have a precise estimation of market risk.

Early developments of such liquidity-adjusted VaR identified substantial underestimation of costs. For example, Bangia *et al.* (1999) find that total risk is under-estimated by 25-30% in emerging market currencies for a daily VaR, Le Saout (2002) reports that the bid-ask spread component can represent over 50% of total risk for illiquid stocks, and Lei and Lai (2007) find that 30% of total intraday risk can be attributed to illiquidity in small-price stocks. As for the price impact beyond the spread, omitting liquidity costs can underestimate total market risk by 2-21% (Francois-Heude and Van Wynendaele, 2001), 11-30% for a 30 minute liquidity adjusted VaR (Giot and Gramming, 2005), 11% for low capitalization stocks (Angelidis and Benos, 2006) and 25% for a 10 day 99% VaR when trading large positions (Stange and Kaserer, 2008c).

Thus, the purpose of this thesis is to apply liquidity proxies from the microstructure literature based on daily data to estimate the liquidity cost within a monthly parametric VaR model. It is our hypothesis that the use of such proxies within a VaR framework will be effective in estimating potential loss for US stocks including liquidity costs for a given probability. Such a model would be a simple yet a significant improvement in evaluating market risk and an important step in introducing microstructure effects within a risk management framework. To date, liquidity cost proxies within a VaR framework include: the daily bid-ask spread (Bangia *et al.*, 1999), the information contained in the limit order book (Francois-Heude and Van Wynendaele, 2001; Giot and Graming, 2005; Stange and Kaserer, 2008c), price-quantity functions (Berkowitz, 2000; Cosandey, 2001; Jarrow and Protter, 2005; Angelidis and Benos, 2006), and the cost associated with the optimal execution which aims to limit transaction costs and volatilities.

Because liquidity is composed of a variety of complex aspects such as depth, breadth, resilience, immediacy and tightness, [1] the liquidity risk measures developed so far provide information on only some of those aspects making them hard to evaluate relative to a common benchmark. Bhyat (2010) finds that no one measure is capable of tracking all aspects of liquidity risk and that instead practitioners should use a variety of tools to evaluate liquidity risk. However, for practical purposes, models need to be

---

[1] Depth refers to the number of shares available for trade at a particular price, breadth to the number of orders available and their size, resiliency to how easily a market can absorb shocks, immediacy to how quick one can enter and exit a trade, and tightness to how little the cost is for turning around a trade (Bhyat, 2010).

parsimonious and tractable which is why they often focus on transaction costs which provide a clear framework from which to address liquidity related problems (Loebnizt, 2006; Bhyat 2010). Furthermore, the frequency at which spread-based measures are quoted in the liquidity literature and the continued evidence that they are priced in asset returns and correlated with other measures of liquidity seem to indicate that they are the best measures of depth available (Amihud and Mendelson, 1986; Chordia, 2008; and Bhyat, 2010).

In this thesis, we examine the effective spread cost proxies which according to Stange and Kaserer (2008b) and Loebnitz (2006) are the best estimates of transaction costs and price impacts. Other simple liquidity proxy such as ILLIQ by Amihud (2002) exist but they are considered to be cost per volume proxies which measure price impact (or resiliency) as the slope of the price function at a moment in time (Goyenko 2009). The most notable objection to the Amihud illiquidity ratio is that it only approximates the price impact and does not actually measure the cost of trading (Bhyat, 2010). Since such proxies often do not capture the right scale of their benchmarks (Holden, 2009), they are not as useful in a VaR framework.

Thus, the proxies examined in this thesis include the 'Roll Proxy' of Roll (1984), Zeros by Lesmond, Ogden and Trzcinka (1999), the Holden, Fong and Trzcinka (2011, FHT henceforth) proxy and the Shane and Schultz (2012) High-Low Spread proxy (HLS henceforth) . This choice is based on not only their parsimony and calculation feasibility in the absence of intraday data but the evaluation of their accuracy by Lesmond, Ogden and Trzcinka (1999), Hasbrouck (2009), Holden (2009), Fong, Holden and Trzcinka

(2011), and Shane and Schultz (2012). The general conclusion of these studies is that transaction costs can be reasonably measured using proxies computed from daily data with certain proxies performing better than others in terms of scale and correlation. According to this literature, the FHT and Zeros proxies perform best as substitutes for percent effective spread, percent quoted spread, percent realized spread or percent price impact around the world (Fong *et al.*, 2011), while the Roll proxy performs fairly well overall by being highly correlated with the effective spread, performs better than the other proxies for the 2001-2005 sub period and has the least mean bias when compared to intraday benchmarks (Goyenko, 2009). As for the HLS proxy, Shane and Schultz (2012) find it has the highest cross sectional and stock-by-stock time-series correlation with their TAQ calculated effective spreads than the other estimators they study(Roll, 1984; Lesmond *et al.,* 1999; and Holden, 2009).

In order to test the accuracy of each liquidity proxy in a VaR framework, we first generate an unadjusted parametric monthly VaR model based on the current literature to which we add the estimated liquidity cost based on each of the selected proxies. We then back-test these augmented VaR models against liquidity-adjusted returns using the "true" effective costs calculated from intraday data as provided by the Hasbrouck website. This allows for an evaluation of the accuracy of each liquidity proxy in estimating liquidity risk within the VaR model and to determine which performs best overall. This appears to be the first attempt at introducing liquidity proxies into a VaR model.

A major finding of this thesis is that the FHT, Roll and HLS proxies are indeed good estimators of effective costs making them appropriate additions to a VaR model, while the Zero proxy performs very poorly due to its overestimation of effective costs. With high correlations and similar distributional properties relative to the Hasbrouck effective costs, the FHT proxy performs best when back-tested against liquidity-adjusted returns relative to all other proxies. Although both the Roll and HLS proxy can be highly correlated with the effective spread, the Roll measure tends to overestimate liquidity cost while the HLS proxy often underestimates it. We also find that a model's performance varies significantly depending on the volume traded of a particular stock.

The remainder of the thesis is organized as follows. An overview of current models available in the liquidity-adjusted VaR literature is presented in section two. Section three details the methodology and its implementation for calculating the unadjusted parametric VaR model along with a description of each liquidity proxy. Section four describes the data sample and provides some summary statistics. Section five reports and analyzes the results of an empirical back-test. Section six concludes the thesis.

2. LITERATURE REVIEW

2.1 Liquidity-adjusted Value-at-Risk Models

Models developed for liquidity-adjusted VaR can be divided into two groups. The first uses direct liquidity costs such as the bid-ask spread or order-size-dependent weighted spread. Bangia *et al.* (1999) add an ad hoc measure of liquidity cost to a regular VaR framework, where liquidity cost is estimated as the worst quoted daily bid-ask spread based on its historical empirical distribution. Although this methodology is the simplest of the various models, it assumes that any position can be traded at the quoted spread. It does not consider the price impact of larger positions or the fact that assets can be traded within the spread. In contrast, Francois-Heude and Van Wynendaele (2001) use limit order book information on the Paris Stock Exchange to interpolate the price impact of a large order size beyond their available data to significantly improve their estimates of liquidity cost. Giot and Graming (2005) calculate the weighted spread from the order book using intraday bid and ask prices valid for the immediate trade of any volume of interest using data from the automated auction system Xetra which is employed at various trading venues in Europe. Although its estimates are very accurate, this method requires the availability of a transparent limit order book that is not always available for many exchanges. Ernst *et al.* (2008) use a similar model to Bangia *et al.* (1999) but apply a Cornish Fisher approximation to determine percentiles instead of taking them from the historical empirical distribution to account for skewness and kurtosis. Although this is a slight improvement in terms of distribution estimation, this model only uses the inside bid-ask spread to capture the liquidity component. Finally, Stange and Kaserer (2008c) improve on the model further by using the weighted spread of the order book provided by Deutsche Borse AG which once again is not available for all exchanges.

The second group of studies focuses on indirect risk measures of liquidity determined using price-quantity functions estimated from transaction data. Berkowitz (2000) estimates price impact from past trades using a times series of transaction prices in a linear regression while controlling for other risk factors. Although theoretically more accurate as it includes price impact, the procedure assumes a linear price impact function which may not necessarily be valid. Furthermore, this methodology can be data intensive because intraday data is required to calculate the price impact cost from single trades. Cosandey (2001) improves the model by estimating a concave price impact function from volume data by making the price a function of the number of shares traded. However, the curvature parameters in this function can be difficult to estimate. Jarrow and Protter (2005) apply a similar model as Berkowitz (2000) but instead use sample data from crises transactions to derive a price impact coefficient which improves the timeliness of the model. Angelidis and Benos (2005) develop an implied liquidity cost model from its underlying drivers by combining an inventory model of a market maker with a fundamental model of information asymmetry which yields an implied spread. While this model is appealing for its use of an implied spread from the underlying factors, it is data intensive and relies heavily on structural assumptions.

Various models based on optimal execution also exist. Almgren and Chriss (2000) implement a VaR model with an optimal strategy to minimize volatility risk and transaction costs that arise from permanent and temporary market impacts. Such models are not only very computationally intensive but they depend on parameters which can be difficult to estimate. Furthermore, they assume that the position is liquidated over a certain period of time and not instantaneously against the order book.

Although a valid assumption during times of normal market conditions where a trader wants to limit price impact, it is more tenuous in times of crisis where a trader would be more likely to sell off a position faster if the trader possesses important information or needs to liquidate a position due to a margin call. Furthermore, the marginal gain from a lower cost from delaying a transaction balances the marginal loss due to price risk if liquidity prices are efficient and a trader's risk aversion is greater than or equal to that of the market (Stange and Kaserer, 2008b).

Research also implements such models within a portfolio perspective. Botha (2008), for example, devises a simple way to implement the Jarrow and Subramanian (1997) L-VaR model within a portfolio framework. Although portfolio implementation is the ultimate purpose of any VaR model, this thesis deals with defining a new model for individual liquidity adjusted VaR models and leaves a portfolio implementation for further research.

To summarize, each of these models has its merits and drawbacks. While the first group is easier to implement, the accuracy of their liquidity cost estimates increases with the use of intraday or order book data that is not always available for certain products or even markets. While those in the second group can be more accurate, these models are data intensive and rely heavily on parameters which often are unknown and notoriously difficult to gage.

2.2 Liquidity Proxies

Five studies examine the performance of the available percent cost and cost per volume liquidity proxies. Lesmond, Ogden and Trzcinka (1999) test the relation of three annual percent cost proxies to the annual quoted spread as computed from daily closing quoted spreads for the US. Hasbrouck (2009) assesses the relation of three annual percent cost proxies and one annual cost per volume proxy to a benchmark effective spread and the slope of the price function lambda as computed from high frequency US trade and quote data. Goyenko, Holden and Trzcinka (2009) test how nine annual and monthly percent cost proxies are related to annual and monthly percent cost benchmarks computed form high frequency US trade and quote data as well as 12 cost per volume proxies against their respective benchmarks. Fong, Trzcinka and Holden (2011) perform a similar analysis but with eight percent cost proxies and eleven cost per volume proxies computed on a monthly basis against five benchmarks for forty three exchanges in both developed and emerging countries. Finally, Shane and Schultz (2012) compare the performance of the HLS proxy by comparing it to monthly TAQ effective spreads from 1993 through 2006 along with three other unadjusted low frequency monthly effective spread estimators.

Routledge and Zin (2009) explore theoretically how uncertainty can increase bid-ask spreads and reduce liquidity. They focus on financial derivatives in which trades must rely on an empirical model for the stochastic cash flow process of an underlying security making trading intrinsically model dependent. By specifying preferences that explicitly incorporate model uncertainty in a simple market making setting they find that uncertainty can have an impact on liquidity.

3. SAMPLE AND DATA

For the purpose of testing the models, we use the Hasbrouck (2006) sample which consists of randomly selected companies which are available in both the CRSP and TAQ data bases for the years 1993 to 2003. For each year, 250 companies are selected at random from a population of stocks that are ordinary common shares, in the CRSP database on the last trading day of the year, and did not change trading venue, ticker symbol or CUSIP identifier after August of the year.

For each year, a sample of five years of daily historical returns, end of day and high and low as well midpoint prices, and volumes are used to estimate monthly forecasted returns, return variances, and each liquidity proxy and its respective empirical distribution. Deleting companies without at least 5 years of continuous returns reduces our sample to an average of 145 companies or about 226,032 data points per year. The final sample has an average of 1,748 monthly observations per year for a grand total of 24,888 monthly observations for 1410 different companies over the ten years examined herein.

Table 1 reports basic descriptive statistics of daily returns (RET) for each six year sample used in estimating forecasted returns and variances for the years 1993 to 2003.[2] The 1988-1993 samples, for example, are used to estimate 1993 using a five year rolling sample. Our sample sizes vary from a minimum of 189,829 and a maximum of 330,795 observations or about 1,309 to 2,281 observations per company. Distributional characteristics change depending on the sample in question where skewnesses, kurtoses and standard deviations are at their highest in the 1993-1998 and 1996-2001 samples. This is most likely due to the Dot-com bubble/crash and September 11 attack in 2001, and the Russian Bond default and LTCM crisis which ensued in 1998. Note that none of the return distributions pass the Jarque-Bera test implying non-normal distributions which we adjust for in our volatility estimations using the Cornish-Fisher expansion.

4. METHODOLOGY

4.1 VaR Approach

We use the liquidity-adjusted VaR (L-VaR) framework first introduced by Bangia *et al.* (1998) given by:

$$L - VaR = 1 - \exp(z_\alpha \times \sigma_r) + \frac{1}{2}(\mu_s + \hat{z}_\alpha(s) \times \sigma_s) \quad (1)$$

[2] All tables and figures are presented after the references.

The regular parametric VaR model is given by the first two terms on the right-hand-side (RHS) of equation (1) where σ_r is the volatility of daily log returns and z_α is the desired percentile. The VaR is then augmented with a time-varying bid-ask spread by deducting the cost of half the worst bid-ask spread (one way transaction cost) as determined by μ_s (the mean of the spread), σ_s (the volatility of the spread) and $\hat{z}_\alpha(s)$(the empirically estimated percentile of the spread distribution).

Using the empirical percentiles enables the formulation to avoid any distortions from the non-normality in spreads. The liquidity risk can be calculated as the mean-variance estimated price return percentile and the empirically estimated spread percentile if continuous mid-spreads are assumed to be normally distributed with a daily mean of zero. The estimates of the model improve if the worst spread is deducted from the worst price and the non-normality in the return distribution is accounted for by implementing the Cornish and Fisher (1937) expansion which adjusts the standardized percentiles to account for higher-order moments according to Ernst (2008) and Loebnitz (2006).

Because spread distributions may not be normal (Bangia *et al.*, 1998), we use the empirical distribution of each of the liquidity proxies to determine their mean, standard deviation and percentiles.[3] Hence the model subsequently tested is:

---

[3] The accuracy of various methodologies to determine spread distributions for forecasting has yet to be tested in the literature and is beyond the scope of this thesis.

$$L - VaR = 1 - \exp\left(\mu_r + z_{\alpha cf}(r) \times \sigma_r\right) \times \left(1 - \frac{1}{2}\left(\mu_{Proxy} + z_\alpha(Proxy) \times \sigma_{Proxy}\right)\right) \quad (2)$$

Where μ_r is the expected monthly log return, $z_{\alpha cf}(r)$ is the Cornish-Fisher expansion-adjusted percentile of the return distribution, while $\mu_{Proxy},$ $z_\alpha(Proxy)$ and σ_{Proxy} are the empirically estimated mean, percentile and standard deviation of one of the following liquidity proxies: FHT, Roll, Z and HLS.

For the purpose of calculating our unadjusted VaR (first two RHS terms of equation 2), given by:

$$VaR = 1 - exp(\mu_r + z_{\alpha cf}(r) \times \sigma_r) \quad (3)$$

we use a 95% confidence level for the estimation of each stock's worst expected loss as Contreras (2010) argues results in a more robust VaR model. To forecast monthly expected returns and volatilities from daily data, we use a GARCH (1, 1) as developed by Bollerslev (1986). It involves the joint estimation of a conditional mean (AR(1)[4]) and conditional variance equation where the variance is calculated by maximizing the log likelihood function of the alpha (α) and beta (β) and where standard errors are computed using the robust method of Bollerslev-Woodridge. More formally:

$$r_t = c_0 + c_1 r_{t-1} + \varepsilon_t \quad (4)$$

[4] We implemented a simple AR(1) model for the conditional mean equation as Angeledis (2004) finds that the mean process specification plays no important role. This is based on his implementation of several volatility models under three different distributional assumptions of returns and four historical sample sizes to estimate the 95% and 99% one day VaR for five completely diversified index Portfolios.

$$h_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}^2 \quad (5)$$

where r_t is the daily log return, $\varepsilon_t=z_t h_t$ and z_t is a sequence of IID distributed random variables with mean zero and unit variance, c_0 and c_1 are the coefficients for our AR(1) model, and h_t is our conditional variance.

While various conceptually more accurate GARCH models exist, there is yet no consensus on which model is the most appropriate for each type of financial asset, sample frequencies, performance tests, target probabilities or sub periods. Overall the basic GARCH (1-1) model seems to be the most versatile and widely accepted model of choice and is supported by various studies. These include Ederington (2005) who finds that the GARCH is a more accurate measure of volatility than the exponentially weighted moving average (EWMA) model and Brownless *et al.* (2012) who find that, although the TARCH model perform best overall when forecasting variances for one-day ahead, the difference between asymmetric and symmetric GARCH(1,1) becomes insignificant over a one-month horizon.

Chrétien (2008) finds that the unconditional VaR models generally underperform conditional VaR models with respect to the independence test as they underestimate the frequency of consecutive VaR violations. He also finds that the use of daily rather than monthly data improves the performance of monthly VaR calculations which he attributes to better volatility estimation when using higher frequency data. This is further supported by various studies such as Akgiray (1989) with daily CRSP value-weighted and equal-weighted indices returns, Brailsford (1996) with daily Australian stock returns, Niguez (2008) with daily Eurostoxx 50 index returns, and Fligewski (1997) with daily S&P 500 returns. Furthermore, based on their survey of 93 academic works on volatility forecasting in financial markets, Huang and Granger (2003) find that papers are notoriously difficult to compare since they are prepared for different reasons and use different data sets and estimation techniques. Indeed, model performance may be dependent on the financial product, region and time period studied.

We then estimate the monthly variances following the methodology recommended by Brailsford (1996) and Ederington (2005) where a daily s-step ahead forecast, $\hat{h}_{t+s}$, can be formed based on the GARCH (1,1) model as follows

$$\hat{h}_{t+s} = \hat{\omega} \sum_{i=0}^{s-2} (\hat{\alpha}_1 + \hat{\beta}_1)^i + (\hat{\alpha}_1 + \hat{\beta}_1)^{s-1} \hat{h}_{t+1} , \qquad s = 1,2, \dots , N_T \tag{6}$$

where N_T is the last forecasted day. Monthly volatility forecasts are then formed by aggregating the s-step ahead daily forecasts across trading days in each month by summing the within-month daily variance forecasts as follows:

$$\hat{\sigma}_T^2 = \sum_{s=1}^{N_t} \hat{h}_{t+s} \tag{7}$$

where a given month has 22 days (N_T) .

Initially the GARCH model is estimated over a five-year period from 1983 to 1987 where the parameter ω, α_1 and β_1 are estimated and the s-step ahead forecast is calculated for the following month. The summation of the s-step-ahead forecasts gives us the forecasted monthly variance for that month. Much like Engle *et al.* (1993), Angeldis (2004) finds that the impact of sample size on model performance depends on the series under examination, and that a sample of 1000 to 2000 observations performs best for US stocks. Both studies conclude that a restriction on the length of the estimation sample may be beneficial as it limits the use of outdated data. Our estimation sample of five years or about 1320 days is consistent with their findings. The estimation sample is then rolled one month forward and the parameters and forecasted variances are re-estimated. The same procedure is used to estimate forecasted monthly returns where the 22 days ahead daily log return forecasts are summed to obtain a monthly value.

Although the square root of time rule ($\hat{\sigma}_T = \hat{\sigma}_{t+1} \times \sqrt{N_T}$) is a popular shortcut to convert daily volatility into longer horizons it has been criticized by various papers such as Christoffersen *et al.* (1998), Blake *et al.* (2000) and Danielssen *et al.* (2006) for being restricted to the Gaussian distribution and that its accuracy tends to decline significantly with an increasing horizon. Although various adjustments are proposed in the literature such as the time scaling quantiles of return distribution (Danielssen *et al.* , 2006) or the aggregation formula (Drost and Nijman, 1993), we only use both the square root of time rule and the sum of forecasted variance where the latter should be more accurate as it reflects the reversion process of the variance estimation.

4.2 Liquidity Proxies

The following are the selected liquidity proxies that are used within our VaR framework. As previously mentioned, they are selected because they can be estimated using only daily data.

4.2.1 Roll Measure

Roll (1984) estimates effective spreads as the serial covariance of the changes in prices given by:

$$V_t = V_{t-1} + \varepsilon_t \tag{8}$$

Where V_t is the fundamental value of a stock on day *t*; and e_t is the mean-zero serially uncorrelated public information shock on day t. The last observed traded price on day t, P_t, is given by:

$$\mathrm{P}_t = \mathrm{V}_t + \frac{1}{2}\mathrm{S}\,\mathrm{Q}_t \tag{9}$$

Where S is the effective spread and Q_t is a buy/sell indicator of the last trade where +1 is a buy and -1 is a sell. Taking the first differences of the closing prices yields:

$$\Delta P_t = \frac{1}{2} S \Delta \mathrm{Q}_t + \varepsilon_t \tag{10}$$

where Δ is the change operator. Solving for S yields:

$$Roll = S = 2\sqrt{-Cov(\Delta P_t, \Delta P_t - 1)} \tag{11}$$

The result is undefined if the sample serial covariance is positive. In this case, S is set either to zero (i.e., treated as missing) or estimated by arbitrarily multiplying the covariance by negative one. Following the methodology in Goyenko *et al.* (2009), we replace positive covariances with zeros. The covariance of the change in the price of the stock for the current and previous day is calculated using the past 22 days. Although there are adjustments to this methodology proposed by Holden (2009) and Hasbrouck (2004), we only implement its original form within a VaR model.

4.2.2 Zeros Measure

Lesmond *et al.* (1999) estimate liquidity by calculating the proportion of zero return days defined as:

$$Z = (\#\ of\ days\ with\ zero\ returns)/T \tag{12}$$

where Z is calculated by simply dividing the number of days with zero returns in a month by the total number of trading day within that month (T = 22 days herein).

4.2.3 FHT Measure

The FHT proxy developed by Fong, Holden and Trzcinka (2010) is a simplification of the LOT measure of Lesmond *et al.* (1999). If transaction costs are assumed to be symmetric, S/2 and –S/2 are the percent transaction costs of buying and selling the same stock, respectively, where S is the round-trip transaction cost. The observed returns R on an individual stock are given by:

$$R = R^* + \frac{S}{2} \quad when\ R^* < -\frac{S}{2}$$

$$R = R^* \quad when \quad -\frac{S}{2} < R^* < \frac{S}{2} \tag{13}$$

$$R = R^* - \frac{S}{2} \quad when\ \frac{S}{2} < R^*$$

where the unobserved true return R^* of an individual stock is assumed to be normally distributed with mean zero and standard deviation σ. Hence, the theoretical probability of a zero return is given by

$$N\left(\frac{S}{2\sigma}\right) - N\left(-\frac{S}{2\sigma}\right) = Z \tag{14}$$

where Z is the empirically observed frequency of a zero return (as calculate previously). Solving for S then yields the following formula:

$$FHT = S = 2\sigma N^{-1}(\frac{1+Z}{2}) \tag{15}$$

where $N^{-1}(\bullet)$ is the inverse function of the cumulative normal distribution.

4.2.4 HLS Measure

The HLS measure of Shane and Schultz (2012) attempts to estimate bid-ask spreads from daily high and low prices based on two assumptions. The first is that daily high (low) prices are almost always buyer (seller) initiated trades, and hence that the high-to-low price ratio for a day reflects both the fundamental volatility of the stock and its bid-ask spread. The second assumption is that the component of the high-to-low price ratio that is due to volatility increases proportionately with the length of the trading interval, while the component due to bid-ask spreads does not. Based on these two assumptions, they infer that the sum of the price ranges over two consecutive days reflects two days' volatility and twice the spread and that the price range over one two day period reflects two day volatilities and one spread. Based on these observations, they derive an estimate of a stock's bid-ask spread as a function of the high-to-low price ratios for a 2 day period and the high-to-low price ratios for two consecutive single days. They derive the following function for the spread:

$$HLS = S = \frac{2(e^{\alpha}-1)}{1+e^{\alpha}} \tag{16}$$

where

$$\alpha = \frac{\sqrt{2\beta}-\sqrt{\beta}}{3-2\sqrt{2}} - \sqrt{\frac{\gamma}{3-2\sqrt{2}}} \tag{17}$$

$$\beta = \sum_{j=0}^{1}\left[\ln\left(\frac{H_{t+j}^{o}}{L_{t+j}^{o}}\right)\right]^2 \tag{18}$$

$$\gamma = \left[\ln\left(\frac{H_{t,t+j}^{o}}{L_{t,t+j}^{o}}\right)\right]^2 \tag{19}$$

and H_t^O and L_t^O are the observed high and low stock prices for day t. To get a spread estimate for a month, we average the spread estimates from all overlapping two-day periods within the month.

4.2.5 Empirical distributions of the liquidity proxies

Because liquidity costs are not normally distributed based on the Jarque-Bera test results reported in table 2, we calculate each liquidity proxy's empirical percentiles with a 95% confidence level. This is the same as the percentile used for our returns within our VaR model. We do this by first finding each proxy's 95% highest value ($\hat{S}_\alpha$), mean (μ_S) and standard deviation (σ_S) using a five-year moving estimation window of monthly data, which is the same time frame as for our GARCH model. The empirical percentile for the desired confidence level is then given by:

$$z_\alpha(Proxy) = \frac{\hat{S}_\alpha - \mu_S}{\sigma_S} \tag{20}$$

Table 2 reports some descriptive statistics for each liquidity proxy by sample that is used in estimating the moving average, standard deviation and empirical percentiles. The distributional characteristics of each proxy are crucial in determining how it will perform within a VaR framework. The proxies need not only to be correlated with the true effective spread but also to reflect its scale and variance. Our sample sizes range from as high as 12,600 observations to as low as 8,403 observations, which represents from about 58 to 86 monthly observations per company. We can observe from table 2 that the Z proxy is much larger than any of the other proxies for all the samples. While the FHT and Roll proxies are fairly similar for the early samples (1989-1993 to 1993-1998), they differ thereafter. Specifically, the FHT estimates consistently diminish relative to the corresponding Roll estimates which also decrease over time but at a lesser extent. The HLS estimates are lowest relative to the other proxies for all studied samples. In the following section, we will examine how the various estimates compare to true effective costs for the years 1993-2003 in order to estimate which estimator is more likely to perform best in our L-VaR models.

4.2.6 Accuracy of each liquidity proxy in determining effective costs

A liquidity proxy's performance in a VaR model depends on how accurately it reflects the scale, distribution and behavior of "true" effective costs. We expect that this accuracy will differ for our four chosen liquidity proxies. Thus, in this section, we test the following null hypothesis:

H_0^1: All four chosen liquidity proxies have the same accuracy in estimating "true" effective costs.

To estimate the accuracy of each proxy in determining effective cost, we compare the estimates generated for each proxy against the actual effective cost calculated from intraday data as provided by the Hasbrouck data set (effCostLog series). [5] These benchmark estimates are based on TAQ quotes where quotes with zero bids or asks, asks greater than five times their corresponding bids, and absolute spreads greater than five dollars were deleted. Only quotes from each stock's primary exchange during regular trading hours were used. All trades with nonstandard settlements or corrections were deleted before they were signed where a trade price above (below) the midspread is presumed to be a buy (sell) order (Hasbrouck, 2006). The effective spread is

---

[5] http://people.stern.nyu.edu/jhasbrou/Research/GibbsEstimates2006/Liquidity%20estimates%202006.htm

calculated using mid-spreads prevailing two seconds prior to the reported trade time. Effective cost outliers above the 95th percentile are removed for months with more than 100 trades. Each effective spread is then weighted by the dollar volume of trade and averaged over the month.

Some of the basic descriptive statistics of the “true” percentage effective spreads, which have been multiplied by two to reflect round trip transactions, and the liquidity proxies for the years 1993-2003 are reported in table 3. We observe that the Roll proxy best reflects the scale of the true effective cost (represented as EFFCOST in table 3). It has a mean of 0.026, standard deviation of 0.0362, and skewness of 4.5614 which are similar to our EFFCOST mean of 0.0254, standard deviation of 0.0365 and skewness of 3.9889. FHT and HLS follow closely behind with slightly lower mean and standard deviation estimates but much larger skewness and kurtosis estimates. The Z proxy once again exhibits distributional properties that vary greatly from EFFCOST.

Table 4 reports a correlation matrix of the true effective costs from Hasbrouck and our four liquidity proxies (FHT, Roll, Z and HLS).[6] We observe that FHT outperforms the other three liquidity proxies with a correlation of 0.73 with EFFCOST. The Roll liquidity proxy is next with a correlation of 0.69, the HLS liquidity proxy third with a correlation of 0.61, and the Z liquidity proxy last with a correlation of 0.45. To this point, the FHT and Roll liquidity proxies appear to be our best candidates for incorporating liquidity costs within a VaR model.

[6] The correlation matrices based on annual results are presented in appendix I.

5. BACKTESTS

5.1 Hypothesis and Methodology

VaR models are only as good as their ability is to predict risk. No matter how advanced or complex the methodology is to estimate future loss, a model is suspect if it constantly over or under estimates risks. Thus, in this section, we test the following null hypothesis:

H_0^2: The addition of our liquidity proxies to a VaR model will not provide better estimates of potential liquidity-adjusted return losses.

Our expectation is that the addition of at least one of our four chosen liquidity proxies to a VaR model will provide better estimates of potential liquidity-adjusted return losses.

A perfectly calibrated model should have an equal percentage of overestimations at the chosen confidence level. Because we are using a confidence level of 95%, we expect that about 5% of our monthly liquidity-adjusted monthly stock returns will exceed our unadjusted VaR(L-VaR) estimates. More formerly:

$$VaR_t > \exp(R_t) - 1 \tag{21}$$

$$L - VaR_t > \exp(R_{net,t}) - 1 \tag{22}$$

Where R_t are monthly log returns (sum of daily log returns within one month) that are adjusted by subracting the true effective costs as calculated by Hasbrouck's effcostlog series (EFFCOST), and $R_{net,t}$are liquidity-adjusted returns given by:

$$R_{net,t} = R_t - EFFCOST_t \tag{23}$$

Two major weaknesses are found in the literature when comparing L-VaR models. The first weakness is that new models with a conceptually more accurate estimate of liquidity cost (usually more computationally complex and data intensive) are tested against clearly inferior liquidity-adjusted or unadjusted models (e.g., Francois-Heude, 2001; Giot, 2005; Berkowitz, 2002; Cosandey, 2002; and Angelidis, 2005). The second weakness is that the returns against which they are tested have the same liquidity adjustment as one of the models in question (Ernst, 2009). Clearly the L-VaR model using the same liquidity adjustment as the adjusted returns will perform best as it has the unfair advantage of using the same proxy used to adjust returns. For our purposes, the L-VaR model using the best liquidity proxy should perform as well as our unadjusted VaR model.

To evaluate the ability of the VaR models to meet the target probability of 5%, three likelihood ratio tests using the methodology developed by Christoffersen (1998) are used. Since the parameters used herein to calculate our L-VaR estimates are backward estimated, the back-testing is indeed out of sample.

The first test is the unconditional coverage test which estimates the proportion of VaR violations relative to the theoretical target probability using:

$$LR_{unc} = 2Log\left[\frac{(1-\frac{n_1}{n_0+n_1})^{n_0}(\frac{n_1}{n_0+n_1})^{n_1}}{(1-\alpha)^{n_0}(\alpha)^{n_1}}\right] \tag{24}$$

where n_1 and n_0 are the number of VaR violations and non-violations, respectively, and α is 5% as determined by our confidence level of 95%. The likelihood-ratio is asymptotically χ^2 distributed with one degree of freedom. If the value of the LR_{unc} statistic exceeds the value of the χ^2 distribution critical value, then the null hypothesis that the number of violations is equal to the theoretical target probability will be rejected.

The second test is the independence test which evaluates whether VaR violations can be predicted depending on the result of the previous day. The proportion of VaR violations should not depend on whether a VaR violation occurred in the previous period. This maximum likelihood test with one degree of freedom is given by:

$$LR_{ind} = 2Log\left[\frac{(1-\frac{n_{01}}{n_{00}+n_{01}})^{n_{00}}(\frac{n_{01}}{n_{00}+n_{01}})^{n_{01}}(1-\frac{n_{11}}{n_{10}+n_{11}})^{n_{10}}(\frac{n_{11}}{n_{10}+n_{11}})^{n_{11}}}{(1-\frac{n_1}{n_0+n_1})^{n_0}(\frac{n_1}{n_0+n_1})^{n_1}}\right] \tag{25}$$

where n_{01} and n_{00} are the number of VaR violations and non-violations following a non-violation respectively, and n_{11} and n_{10} are the number of VaR violations and non-violations following a violation respectively.

The final test examines whether or not the VaR models jointly meet the two preceding conditions. This is evaluated by the following conditional coverage test that is also χ^2 distributed but instead has two degrees of freedom:

$$LR_{cond} = LR_{unc} + LR_{ind} \tag{26}$$

If this test results in a rejection, then the VaR violations are not simultaneously independent of each other and are not of a proportion corresponding to the target probability.

5.2 Backtest Results

5.2.1 L-VaR Backtest

Tables 5 and 6 report the performance of each L-VaR and unadjusted VaR models against adjusted returns and unadjusted returns respectively. Table 5 uses the sum of forecasted variances and table 6 uses the square root of time rule methodology to estimate the monthly volatility. Testing the unadjusted model helps us determine if the lack of performance from the L-VaR is due only to poor liquidity cost estimation or also to estimation.

At the 95% confidence level, the critical values for the LR_{unc} and LR_{ind} tests with one degree of freedom are 3.84, and the critical value for the LR_{cond} test with two degrees of freedom is 5.99. Based on the unadjusted model results in table 5, we observe that out of the 11 years tested the model is not rejected in only 3 years by the unconditional coverage test (1997, 2001, and 2002), 6 years by the independence test

(1993, 1994, 1995, 1996, 2001, and 2003), and only one year by the conditional coverage test (2001). Nevertheless, the model has a total % violation for the entire sample of 4.75% which is close to the 5% theoretical target probability used.

For the liquidity-adjusted models, we first observe that the Z L-VaR model has poor performance in estimating liquidity-adjusted expected losses. Although it is successful in passing the independence test in each year of the sample, it is constantly rejected by the unconditional coverage test. This is further supported by its overall % violation of 0.88% which is far from our targeted probability of 5%. These results are expected as we have already concluded in our liquidity analysis based on table 3 that the Z proxy did not adequately reflect the distributional properties of true effective costs. Indeed, the more a particular proxy overestimates true liquidity cost, the less likely it is to be violated hence failing the unconditional test and "passing" the independence test.

However, the other three L-VaR models show more promise. Among these three models, the % of violations is highest for the HLS L-VaR model, followed by the FHT and Roll L-VaR models. This is consistent with previous findings reported in table 3 where HLS has the lowest liquidity cost estimates followed by FHT and then Roll. If liquidity cost estimates are lower, they are more likely to be exceeded by the true effective costs, resulting in higher levels of violations. All three models perform in a similar fashion as the unadjusted model by not being rejected in 3 out of the 11 years examined by the unconditional test. The FHT and HLS L-VaR models actually fail to be rejected the same three years whereas the Roll L-VaR model is rejected in 2001 but is not in 1999. Clearly this difference is due to the differences in scales between proxies and is by no means indicative of certain models performing better than others in certain years. Out of those three years only the FHT and HLS L-VaR models manage to also pass the independence and conditional tests for the same year passed by the unadjusted model (2001).

The results for the square root of time rule methodology that are presented in table 6 show similar if not marginally better results. Overall the monthly variance estimates are smaller than when we used the sum of daily forecast method as our percentage of violations are greater for each VaR model. The unadjusted VaR model fails to be rejected both in 1994 and 2001 by the conditional coverage test and additionally only in 1997 by the unconditional coverage test. The L-VaR models perform better. Both the Roll and HLS L-VaR models pass the conditional coverage test for the same two years as the unadjusted model. The FHT L-VaR model performs the best failing to be rejected in 3 of the 11 years examined (1994, 1997, and 2001).

Figures 1 and 2 are graphical depictions of the % violations for the sum of forecasted variances and the square root of time methodology respectively. For our models to be accurate, the % of violations should be close to our target probability of 5%. Irrespective of the methodology or liquidity proxy used, we observe periods of time where risk is overestimated such as from 1993 to 1996 where % violations are below 5%, or where risk is underestimated such as from 1998 to 2000 where % violations are above 5%. This pattern is likely due to the large sample used in estimating stock return volatility forecasts. When volatility increases in 1997 and 1998 due to the run on the Thai Bath and the hedge fund breakdowns of LTCM (Bhyat, 2010), the sample is not responsive enough to market changes. This leads to subsequent underestimations of risk for the following periods. The reverse is also true when volatility in the market decreases. For example, the results for 2003 show significant overestimations of risk with much lower % violations than our target probability. This is once again most likely due to our 5 year sample from 1998-2002 which includes much more volatile stock returns than is actually experienced in 2003.

5.2.2 Liquidity-adjustment backtest

A model's weakness cannot only be due to its inability to estimate liquidity cost but also with its inability to correctly forecast monthly returns and volatilities. If the unadjusted VaR model does not perform adequately there is little value in testing the adjusted model which includes the same return and variance estimates. Hence, to determine if our liquidity proxies are able to estimate true liquidity costs within a VaR framework, we test them individually using the same backtesting methodology described previously. However, instead of testing the results of our complete L-VaR models against liquidity-adjusted returns, we now test the liquidity adjustment measure of our models against the "true" effective costs, where a violation constitutes a "true" effective cost greater than the worst expected liquidity proxy cost fora predetermined α. More formally, we test:

$$EFFCOST_t < \frac{1}{2}(\mu_{Proxy} + z_\alpha(Proxy) \times \sigma_{Proxy}) \tag{27}$$

Based on the results reported in table 7, none of the liquidity proxies survives the conditional coverage test. As we concluded previously, this is the case for the Z proxy due to its overestimation of liquidity costs which results in very few violations (0.01%) and consequently large $LR_{unc}$and low LR_{ind} results for each year in the sample. Once again, we see a pattern between the Roll, FHT and HLS proxies where the Roll measure has the fewest violations with a total percentage of 3.52% followed by the FHT and HLS measures with 7.26% and 20.09%, respectively. This is as expected as we previously found that the Roll measure is consistently greater than the FHT and HLS measures resulting in fewer overall violations. The test results show that the Roll measure is not rejected by the LR_{unc} test in 3 out the 11 years tested (years 1994, 1995, and 1996) but fails the LR_{ind}test every year and consequently the LR_{cond} test. At the other end of the spectrum, we have the HLS proxy which underestimates "true" effective costs significantly with an overall violation percentage of 20.09%. The number of violations are so high in all years except 1998 that it renders our denominator and nominators in our $LR_{unc}$and LR_{ind} calculations infinitely small or equal to zero causing our test results to be infinite and subsequently rejected. The FHT proxy performs best with the closest percentage of violations to our target probability of 5% and by also not being rejected in 3 out the 11 years by the unconditional test (years 1997, 1998, and 2003). However, the proxy fails to adapt quickly enough to changes in the variances of liquidity as it is rejected every year by the independence test.

Figure 3 is a graphical depiction of % violations for each liquidity adjustments against true effective costs. The proxies show a pattern where % violations decrease from 1993 to 1998, then increase until 2000 to finally decrease again until 2003. (Note that this is less true for the Roll proxy which becomes more stable from 1998 onwards, which is most likely due to the methodology used in calculating the proxies.) Such patterns are another indication of our model's inability to respond to market changes quickly. As liquidity risk increased in 1997, 1998, 2001 and 2002 the % violations decrease. The reverse is also true when liquidity risk is low in 1999 and in 2000.

Although our results are not conclusive, we observe that that there is some validity in using liquidity proxies to estimate "true" effective costs. This is encouraging in the sense that further research could be done to better estimate its volatility and distribution. In this thesis our variances and means of our liquidity proxies are calculated using an arbitrary rolling sample of 60 observations which may not be optimal for forecasting purposes.

5.2.3 Backtest results sampled by volume

Trading-activity-based measures are parsimonious and widely available measures to evaluate the overall liquidity of an asset. Although they do not capture the cost attributable to liquidation they do serve as an indicator of overall market breadth. While the literature cites a large number of such proxies, volume appears to be the better measure in asset pricing tests (Clochette *et al.,* 2008). For example, Keen *et al*. (2007) find that by regressing returns on 54 NYSE stock portfolios sorted by book-value/market value, size, liquidity and momentum against “factor mimicking” returns with liquidity risk isolated, that the volume traded of a stock shows the most robust pricing effects when compared to other liquidity measures. Fleming *et al.* (2003) find similar results by regressing a stock's five-minute price change against its trading frequency and trading volume amongst other liquidity measures. They conclude that returns are impacted more strongly by trading volumes than by any other trading-activity measures. They also find that trade volume is more highly correlated to other measures of liquidity than proxies such as trade size or quote size. When comparing various liquidity measures, Bhyat (2010) finds not only that the bid-ask spread plays a pivotal role in the overall liquidity of a stock but that it is strongly negatively correlated with volume particularly for more illiquid stocks. As such, volume traded can be used as an indicator to evaluate a stock’s liquidity.

When backtesting various L-Var and unadjusted VaR models against realized losses from a simulated trading strategy, which specifically accounts for liquidity-related costs, we find that the standard VaR models do not match the realized P&L as closely for illiquid as for liquid stocks. This stems from the fact that standard VaR models do not address the special needs of illiquid stocks which need to account for liquidity risk. However, we also find that the more illiquid a stock is the less accurate are its proxies rendering the L-VaR models just as poor as their unadjusted counterparts. By segmenting our sample into deciles based on volume traded within a month (1922 observations each), we can observe how trading activity can impact our VaR estimates. Presumably, we should see more significant differences between our unadjusted VaR and L-VaR models for more illiquid stocks as liquidity costs become a larger factor and our proxies less accurate. Results between the two models for more liquid stocks should be fairly similar as liquidity costs become less material.

Table 8 reports the backtesting results for each decile using the sum of forecasted variance method. The unadjusted model performs best in the mid-range (deciles 6 and 7) by not being rejected by the conditional test. Figure 4 shows a clear pattern in % violations between each decile where the number of violations increases from the lower volume traded deciles to the higher ones. Clearly the unadjusted VaR model is biased in that it overestimates risk for low volume traded stocks and underestimates it for higher volume traded ones with a percentage violation of 2.34% and 8.89%, respectively.

We observe the same pattern for the L-VaR models except that overall the % of violations are smaller than for the unadjusted model. However, this is not the case for the HLS L-VaR model for the lowest deciles where the numbers of violations are actually greater than that for the unadjusted model. Because each proxy's calculations are based on different assumptions, their performances will differ depending on the data available or in this case the level of trading volume.

In appendix II we report correlation matrices for each decile for all four liquidity proxies and the "true" effective spreads. For the lower deciles, the FHT measure performs best followed by the Roll, HLS and Z proxy. However, as we move towards the higher trading-volume deciles, the correlation coefficients of the FHT and Roll measures relative to "true" effective costs deteriorate until they are eventually surpassed by the HLS measure in the 5th to 10th deciles. As such the FHT L-VaR model should perform better in lower trading-volume deciles while the HLS L-VaR model should be superior in the higher trading-volume deciles. We observe some support for this conjecture in table 8 for the first decile where the HLS L-VaR model differs mostly from the unadjusted model. However, the HLS L-VaR model generally performs better than the other three models for the other nine deciles where is has the closest % violations to the unadjusted models. Furthermore, the HLS model performs best in the same deciles as the unadjusted model whereas the Roll and FHT models perform best in the higher deciles. This most likely is due to the underestimations of risk by the unadjusted models which is compensated by the L-VaR models' overestimation of liquidity risk.

Table 9 and figure 5 report the same backtesting results but using the square root of time methodology. Results are similar in the sense that we see the % of violations increase from the lower to higher trade-volume deciles. As previously mentioned, the square root of time method underestimates risk relative to the sum of forecasted variance method. This results in greater % violations when compared to the sum of variance method. The best performing deciles for the unadjusted VaR are the 5th, 6th and 7th deciles which are not rejected by the conditional test with values of 3.27, 3.09 and 0.63, respectively. The best performing L-VaR model is now the FHT L-VaR model as it is not rejected by the conditional test for the 5th, 6th, 7th and 8th deciles. It is followed by the Roll , FHT and Z L-VaR models. As for our per annum results, we observe that with the lower risk estimations using the square root of time methodology, models with larger liquidity risk estimations such as the FHT and Roll L-VaR models outperform smaller liquidity risk estimation models such as the HLS L-VaR model.

5.2.4 Liquidity-adjustment backtest for samples sorted by volume

As in section 5.2.2, we now examine how each liquidity proxy performs when compared solely against “true” effective costs when we account for differences in traded volumes. Table 10 and figure 6 report the backtesting results for each liquidity proxy in each of the volume-traded deciles. Much like for our estimation of stock volatility in the unadjusted VaR model, we observe a trend in figure 6 from one decile to another. While our unadjusted VaR model overestimates/underestimates risk for stocks with low/high trading volumes, our liquidity adjustments behave in an opposite fashion by underestimating/overestimating liquidity costs for stocks with low/high trading volumes with % violations decreasing from the lower to higher trading-volume deciles.

The Roll proxy appears to perform best in the lower deciles with 6.15 % of violations in the 2nd decile while the FHT performs best in the 4th and 5th deciles, and HLS in the 7th and 8th deciles. Once again no liquidity proxies pass the conditional test but overall the FHT performs best with a 7.26 % average violation rate. Clearly volume traded has a significant impact on liquidity estimation as well as for return variance forecasting. Depending on the liquidity of a stock, certain proxies will perform better than others. This needs to be taken into consideration when implementing a L-VaR model.

5.2.5 Economic recession snapshot

To further test the validity of our models we examine how they perform during periods of high volatility and uncertainty such as during an economic or financial crisis.

Although a VaR model is most valuable during such periods, it often falls short by failing to adjust to changes in market volatility quickly enough. According to Naes *et al.* (2011), there is a strong relation between stock market liquidity and the business cycle where a drain in liquidity can be observed prior to an economic recession. Hence, our liquidity proxies can become valuable leading indicators of forthcoming market downturns which may not be fully captured by our estimations of future return variances.

As in Naes *et al.* (2011), we also rely on the National Bureau of Economic Research (NBER) to determine periods of economic recessions within our data set. We find that our sample includes one economic recession which lasted from March 2001 to November 2001 or 9 months (1269 observations). Table 11 reports the descriptive statistics for each liquidity proxy and “true” effective costs for that period. As we found previously, the Roll proxy has the closest distributional properties to “true” effective costs (EFFCOST in table 2) with a mean of 0.0246 and standard deviation of 0.0347 when compared to 0.0208 and 0.0317 for EFFCOST. However based on the correlation matrix for this recessionary period tabulated in table 12, FHT has the highest correlation (0.70) followed by Roll, HLS and finally Z with the “true” effective costs.

The backtesting results in table 13 also show a preference for the FHT with the FHT L-VaR model being the only model to not be rejected using both the sum of variance or Square Root of time methodologies. The square root of time methodology underestimates risk relative to the sum of variance methodology with a greater % of violations. This is consistent with our observation in section 5.2.1 when analyzing the entire sample although for the full time period we concluded that the square root of time rule was slightly better with the % of violations being closer to our target probability of 5%. However, not surprisingly, we find that the model with the overall highest variance estimations perform best during a period of economic recession which is often associated with increased market volatilities. When each liquidity proxy's worst expected loss with a confidence level of 95% are tested individually against the "true" effective costs (results in table 14), they are all rejected by the conditional coverage test, and only the FHT proxy is not rejected by the unconditional coverage test.

6. CONCLUSION

Although various methodologies have become the standard in risk management for their simplicity and ease of use, no one needs to abide by a particular model as long as the chosen model passes a back-test according to the Basel regulations. However, constructing a parsimonious (or accurate) VaR model possesses various challenges. This thesis examined whether a parsimonious but highly relevant Monthly VaR model using GARCH(1, 1) variances based on daily data and adjusted for non-normality using the Cornish and Fisher expansion could be improved by adding a liquidity cost component based on daily data. Although the results were not conclusive, we found that the FHT and Roll proxies remain fairly good gages of effective liquidity costs with similar scale and relatively high correlations to "true" effective costs based on intraday day considering their simplicity in terms of calculation and wide applicability. We also found that a model's performance varies significantly depending on the volume traded of a particular stock. When sampling for different levels of volume traded, we found that our unadjusted VaR model overestimated/underestimated return volatilities while our liquidity adjustments underestimated/overestimated liquidity costs for low/high volume-traded stocks. These behaviors need to be accounted for when implementing unadjusted and L-VaR models.

There are various topics for further research. First, there appears to be very little research on the distribution of liquidity costs and their various proxies. Because liquidity cost distributions are often non-normal, more research is required on their impact on the estimation if outlier events or the choice of confidence levels. More research should also focus on forecasting liquidity costs. As a first attempt in including a liquidity proxy within a VaR framework, we estimated future liquidity costs using empirical percentiles of each proxy based on arbitrary moving averages and standard deviations. Further research could be done to determine optimal sample size and individual observation weighting schemes. Furthermore, many other liquidity (particularly price-impact) proxies could be incorporated to measure price changes due to broken-lot sales or purchases of a large position.

REFERENCES

Akgiray, V. (1989). Conditional heteroscedasticity in time series of stock returns: Evidence and forecasts. *The Journal of Business* 62:1, 55-80.

Almgren, R. and Chriss, N. (2000). Optimal Execution of portfolio transactions. *Journal of Risk* 3:2, 5-39.

Amihud, Y. and Mendelson, H. (1986). Liquidity and stock returns. *Financial Analysts Journal* 42:3, 43-48.

Amihud, Y. (2002). Illiquidity and stock returns: Cross-section and time-series effects. *Journal of Financial Markets* 5:1, 31-56.

Angelidis, T., Benos, A. and Degiannakis, S. (2004). The use of GARCH models in VaR estimation. *Statistical Methodology* 1:1, 105-128.

Angelidis, T. and Benos, A. (2006). Liquidity adjusted value-at-risk based on the components of the bid-ask spread. *Applied Financial Economics* 16:11, 835-851.

Angelidis, T., Benos, A., and Degiannakis S. (2007). A robust VaR model under different time periods and weighting schemes. *Review of Quantative Finance and Accounting* 28, 187-201.

Bangia, A., Diebold ,F. X., Schuermann, T. and Stroughair, J. D. (1998). Modeling liquidity risk with implications for traditional market risk measurement and management. Working paper, Financial Institutions Center at The Wharton School.

Bangia, A., Diebold , F. X., Schuermann, T. and Stroughair, J. D. (1999). Liquidity on the ouside. *Risk* 12, 68-73.

Berkowitz, J. (2000). Incorporating liquidity risk into value-at-risk models. SSRN Working Paper.

Bhyat, A. (2010). An examination of liquidity risk and liquidity risk measures. Thesis, University of Cape Town.

Blake, D., Cairns, A. and Dowd, K. (2000). Extrapolating VaR by the square-root Rule. *Financial Engineering News* 17, 3-7.

Bollerslev, T. (1986). Generalized autoregressive conditional heteroscedasticity. *Journal of Econometrics* 31, 307–327.

Bollerslev, T. and Wooldridge, J. (1992). Quasi-maximum likelihood estimation and inference in dynamic models with time-varying covariances. *Econometric Reviews* 11:2, 143-172.

Botha, M. (2008). Portfolio liquidity-adjusted value-at-risk. *South African Journal of Economic and Management Sciences* 11: 2, 203-216.

Brailsford, T.J. and Faff, R.W. (1996). An evaluation of volatility forecasting techniques. *Journal of Banking & Finance* 20:3, 419-438.

Brownlees, C., Engle, R.F. and Kelly, B. (2012). A practical guide to volatility forecasting through calm and storm. *The Journal of Risk* 14:2, 3-22.

Brunnermeier, M. K., and Pedersen, L. H. (2009). Market liquidity and funding liquidity. *The Review of Financial Studies* 22:6, 2201-2238.

Campbell, A. (2010). What does VAR mean in 2010? *Risk* 23:4, 24-28.

Chollete, L., Naes, R. and Skjeltorp, J. A., (2008). The risk components of liquidity. SSRN Working Paper.

Chordia, T., Roll, R. and Subrahmanyam, A. (2008). Liquidity and market efficiency. *Journal of Financial Economics* 87, 249–268.

Chrétien, S., and Coggins, F. (2010). Performance and conservatism of monthly FHS VaR: An international investigation. *International Review of Financial Analysis* 19:5, 323-333.

Christoffersen, P. (1998). Evaluating interval forecasts. *International Economic Review* 39, 841-862.

Christoffersen, P., Diebold, F.X. (2000). How relevant is volatility forecasting for financial risk management? Working paper, Wharton University of Pennsylvania 39, 841-862.

Contreras, P. (2010). Is VAR a useful tool in volatile markets? *Risk* 23:10, 66-9.

Cornish, E. A. and Fisher, R. A. (1937). Moments and cumulants in the specification of distributions. *Review of the International Statistical Institute* 5, 307-320

Cosandey, D. (2001). Adjusting value at risk for market liquidity. *Risk* (October), 115-118.

Daníelsson, J. and Zigrand, J. (2006). On time-scaling of risk and the square-root-of-time rule. *Journal of Banking & Finance* 30:10, 2701-2713.

Drost, F.C. and Nijman, T.E. (1993). Temporal aggregation of GARCH processes. *Econometrica* 61:4, 909-927.

Ederington, L. H. and Guan, W. (2005). Forecasting volatility. *The Journal of Futures Markets* 25:5, 465-490.

Engle, R. F. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. *Econometrica* 50:1, 987-1007.

Ernst, C., Stange, S. and Kaserer ,C. (2008). Accounting for nonnormality in liquidity risk. CEFS working paper 14.

Ernst, C., Stange , S. Stange and Kaserer, C. (2009). Measuring market liquidity risk: Which model works best? SSRN Working Paper.

Figlewski, S. (1997). Forecasting volatility. *Financial Markets, Institutions & Instruments* 6:1, 1-88.

Fleming, J.M., (2003). Measuring treasury market liquidity. *Federal Reserve Bank of New York, Economic Policy Review,* Working Paper 9:3, 83-108.

Fong, K. Y. L., Holden, C.W. and Trzcinka, C. (2011). What are the best liquidity proxies for global research? SSRN Working Paper.

Francois-Heude, A. and Van Wynendaele, P. (2001). Integrating liquidity risk in a parametric intraday VaR framework. SSRN Working Paper.

Giot, P. and Grammig, J. (2005). How large is liquidity risk in an automated auction market? *Empirical Economics* 30:4, 867-887.

Goyenko, R., Holden, C., Trzcinka C. (2009). Do liquidity measures measure liquidity? *Journal of Financial Economics* 92, 153-181.

Hasbrouck, J. (2006). Trading costs and returns for US equities: The evidence from daily data. Preleminary draft.

Hasbrouck, J. (2009). Trading costs and returns for US equities: The evidence from daily data. *Journal of Finance* 64, 1445–1477.

Holden, C. (2009). New low-frequency liquidity measures. *Journal of Financial Markets* 12, 778-813.

Jarrow, R. A. and Protter, P. (2005). Liquidity risk and option pricing theory. SSRN Working paper.

JP Morgan (1996). RiskMetrics. Technical document, J.P.Morgan.

Keene, M. A. and Peterson, D. R. (2007). The importance of liquidity as a factor in asset pricing. *The Journal of Financial Research 30:*1, 91-109.

Kupiec, P. (1995). Techniques for verifying the accuracy of risk management models. *The Journal of Derivatives* 3, 73-84.

Lazaregue-Bazard, C. (2010). Exceptions to the rule. *Risk* 23:1, 106-8.

Le Saout, E. (2002). Incorporating liquidity risk in VaR models. SSRN Working paper.

Lei, C. C. and Lai, R. N. (2007). The role of liquidity in value at risk: The case of Hong Kong. SSRN Working paper.

Lesmond, D., Ogden J. and Trzcinka C. (1999). A new estimate of transaction costs. *Review of Financial Studies* 12, 1113-1141.

Loebnitz, K. (2006). Market liquidity risk: Elusive no more - defining and quantifying market liquidity risk. Diploma thesis, University of Twente.

Næs, R., Skjeltorp, J. A. and Arne Ødegaard, B. (2011). Stock market liquidity and the business cycle. *The Journal of Finance* 66:1, 139-176.

Ñíguez, T. (2008). Predicting the monthly volatility of the EuroStoxx 50 using data sampled at different frequencies. SSRN Working Paper.

Poon, S.H. and Granger, C.W.J. (2003). Forecasting volatility in financial markets: A review. *Journal of Economic Literature* 41:2, 478-539.

Roll, R. (1984). A simple implicit measure of the effective bid-ask spread in an efficient market. *Journal of Finance* 39, 1127-1139.

Routledge, B.R. and Zin, S.E. (2009). Model uncertainty and liquidity. *Review of Economic Dynamics* 12: 4, 543-566.

Shane A. C. and Schultz, P. (2012). A simple way to estimate bid-ask spreads from daily high and low prices. *The Journal of Finance* 67:2, 719-759.

Stange, S. and Kaserer, C. (2008b). Market liquidity risk: An overview. Unpublished manuscript.

Stange, S. and Kaserer C. (2008c). Why and how to integrate liquidity risk into a VaR-framework. CEFS working paper 10.

Appendix I.Cross sectional correlation matrix of liquidity proxies and true effective spreads on yearly basis

The cross sectional correlations (and p-values) between the four spread estimates and the "true" effective spreads are reported for each year from 1993 through 2003. The sample includes all companies from the NYSE, Amex, and NASDAQ that were randomly selected by Hasbrouck (2006), have at least 60 months of data in CRSP, and have TAQ-based, trade-weighted proportional (%) effective spreads provided by Hasbrouck (2006) as series effcoslog. The effective spreads are averaged across days within a month. Roll Spread is two times the square root of –1 times the autocovariance of daily returns for the month in question. HLS is the equally weighted average of the high-low spread estimator across all overlapping 2-day periods within the month. Z is the % 0 of return days within a month. FHT is the inverse of the cumulative normal distribution function of Z+1 divided by 2 which is then multiplied by 2 times the standard deviation of returns for that month.

Panel A: 1993, 1867 observations					
	EFFCOST	FHT	ROLL	Z	HLS
EFFCOST	1.0000				

FHT	0.8389	1.0000			
	0.0000	-----			
ROLL	0.8245	0.7653	1.0000		
	0.0000	0.0000	-----		
Z	0.4408	0.5710	0.2741	1.0000	
	0.0000	0.0000	0.0000	-----	
HLS	0.7541	0.7715	0.7275	0.3121	1.0000
	0.0000	0.0000	0.0000	0.0000	-----

Panel B: 1994, 1755 observations					
	EFFCOST	FHT	ROLL	Z	HLS
EFFCOST	1.0000				

FHT	0.7933	1.0000			
	0.0000	-----			
ROLL	0.7419	0.6755	1.0000		
	0.0000	0.0000	-----		
Z	0.3665	0.5954	0.1755	1.0000	
	0.0000	0.0000	0.0000	-----	
HLS	0.6482	0.6213	0.6414	0.2452	1.0000
	0.0000	0.0000	0.0000	0.0000	-----

Appendix I. cont'd

Panel C: 1995, 1612 observations					
	EFFCOST	FHT	ROLL	Z	HLS
EFFCOST	1.0000				

FHT	0.8610	1.0000			
	0.0000	-----			
ROLL	0.8386	0.7632	1.0000		
	0.0000	0.0000	-----		
Z	0.3545	0.5310	0.2008	1.0000	
	0.0000	0.0000	0.0000	-----	
HLS	0.7001	0.6362	0.6485	0.2238	1.0000
	0.0000	0.0000	0.0000	0.0000	-----

Panel D: 1996, 1444 observations					
	EFFCOST	FHT	ROLL	Z	HLS
EFFCOST	1.0000				

FHT	0.8453	1.0000			
	0.0000	-----			
ROLL	0.7889	0.7151	1.0000		
	0.0000	0.0000	-----		
Z	0.4347	0.6095	0.2572	1.0000	
	0.0000	0.0000	0.0000	-----	
HLS	0.7089	0.6864	0.6836	0.2756	1.0000
	0.0000	0.0000	0.0000	0.0000	-----

Appendix I. cont'd

Panel E: 1997, 1557 observations					
	EFFCOST	FHT	ROLL	Z	HLS
EFFCOST	1.0000				

FHT	0.8206	1.0000			
	0.0000	-----			
ROLL	0.7055	0.6165	1.0000		
	0.0000	0.0000	-----		
Z	0.4604	0.5971	0.2005	1.0000	
	0.0000	0.0000	0.0000	-----	
HLS	0.7518	0.7036	0.6328	0.2816	1.0000
	0.0000	0.0000	0.0000	0.0000	-----

Panel F: 1998, 1415 observations					
	EFFCOST	FHT	ROLL	Z	HLS
EFFCOST	1.0000				

FHT	0.4736	1.0000			
	0.0000	-----			
ROLL	0.5344	0.6716	1.0000		
	0.0000	0.0000	-----		
Z	0.5052	0.4011	0.2084	1.0000	
	0.0000	0.0000	0.0000	-----	
HLS	0.3646	0.8769	0.6618	0.1472	1.0000
	0.0000	0.0000	0.0000	0.0000	-----

Appendix I. cont'd

Panel G: 1999, 1586 observations					
	EFFCOST	FHT	ROLL	Z	HLS
EFFCOST	1.0000				

FHT	0.5650	1.0000			
	0.0000	-----			
ROLL	0.4216	0.5504	1.0000		
	0.0000	0.0000	-----		
Z	0.3557	0.5930	0.1551	1.0000	
	0.0000	0.0000	0.0000	-----	
HLS	0.3839	0.5737	0.5260	0.1397	1.0000
	0.0000	0.0000	0.0000	0.0000	-----

Panel H: 2000, 1590 observations					
	EFFCOST	FHT	ROLL	Z	HLS
EFFCOST	1.0000				

FHT	0.6147	1.0000			
	0.0000	-----			
ROLL	0.4566	0.5193	1.0000		
	0.0000	0.0000	-----		
Z	0.3992	0.6081	0.1244	1.0000	
	0.0000	0.0000	0.0000	-----	
HLS	0.3928	0.5079	0.4794	0.0801	1.0000
	0.0000	0.0000	0.0000	0.0014	-----

Appendix I. cont'd

Panel I: 2001, 1682 observations					
	EFFCOST	FHT	ROLL	Z	HLS
EFFCOST	1.0000				

FHT	0.7107	1.0000			
	0.0000	-----			
ROLL	0.6467	0.5679	1.0000		
	0.0000	0.0000	-----		
Z	0.4172	0.6407	0.1824	1.0000	
	0.0000	0.0000	0.0000	-----	
HLS	0.4708	0.4627	0.5561	0.0713	1.0000
	0.0000	0.0000	0.0000	0.0035	-----

Panel J: 2002, 2022 observations					
	EFFCOST	FHT	ROLL	Z	HLS
EFFCOST	1.0000				

FHT	0.6871	1.0000			
	0.0000	-----			
ROLL	0.5470	0.5344	1.0000		
	0.0000	0.0000	-----		
Z	0.4230	0.6514	0.1314	1.0000	
	0.0000	0.0000	0.0000	-----	
HLS	0.4230	0.4131	0.4975	0.0167	1.0000
	0.0000	0.0000	0.0000	0.4534	-----

Appendix I. cont'd

Panel K: 2003, 2059 observations					
	EFFCOST	FHT	ROLL	Z	HLS
EFFCOST	1.0000				

FHT	0.7319	1.0000			
	0.0000	-----			
ROLL	0.6046	0.6038	1.0000		
	0.0000	0.0000	-----		
Z	0.5042	0.7356	0.2707	1.0000	
	0.0000	0.0000	0.0000	-----	
HLS	0.4497	0.4777	0.5112	0.1967	1.0000
	0.0000	0.0000	0.0000	0.0000	-----

Appendix II. Cross sectional Correlation matrix of liquidity proxies and true effective spread sampled by volume traded deciles

The cross sectional correlation (and p-value) between the four spread estimates and the "true" effective spreads are reported for each decile based on trade volume. The sample includes all companies from the NYSE, Amex, and NASDAQ that were randomly selected by Hasbrouck (2006), have at least 60 months of data in CRSP, and have TAQ-based, trade-weighted proportional (%) effective spreads provided by Hasbrouck (2006) as series effcoslog. The effective spreads are averaged across days within a month. Roll Spread is two times the square root of –1 times the autocovariance of daily returns for the month in question. HLS is the equally weighted average of the high–low spread estimator across all overlapping 2-day periods within the month. Z is the % 0 of return days within a month. FHT is the inverse of the cumulative normal distribution function of Z+1 divided by 2 which is then multiplied by 2 times the standard deviation of returns for that month.

Panel A: VOL<414, 1922 observations					
Liquidity Proxy	EFFCOST	FHT	ROLL	Z	HLS
EFFCOST	1.0000				

FHT	0.8166	1.0000			
	0.0000	-----			
ROLL	0.7826	0.7076	1.0000		
	0.0000	0.0000	-----		
Z	0.2100	0.5139	0.0210	1.0000	
	0.0000	0.0000	0.3877	-----	
HLS	0.5832	0.5561	0.5112	0.1947	1.0000
	0.0000	0.0000	0.0000	0.0000	-----

Panel B: 414<=VOL<1163, 1922 observations					
Liquidity Proxy	EFFCOST	FHT	ROLL	Z	HLS
EFFCOST	1.0000				

FHT	0.7637	1.0000			
	0.0000	-----			
ROLL	0.7752	0.6865	1.0000		
	0.0000	0.0000	-----		
Z	0.1925	0.4984	0.0580	1.0000	
	0.0000	0.0000	0.0125	-----	
HLS	0.6773	0.5672	0.6239	0.1570	1.0000
	0.0000	0.0000	0.0000	0.0000	-----

Appendix II. Cont'd

Panel C: 1163<=VOL<2438, 1922 observations					
Liquidity Proxy	EFFCOST	FHT	ROLL	Z	HLS
EFFCOST	1.0000				

FHT	0.7754	1.0000			
	0.0000	-----			
ROLL	0.7297	0.6493	1.0000		
	0.0000	0.0000	-----		
Z	0.2143	0.5087	0.0495	1.0000	
	0.0000	0.0000	0.0325	-----	
HLS	0.7262	0.7089	0.7238	0.1800	1.0000
	0.0000	0.0000	0.0000	0.0000	-----

Panel D: 2438<=VOL<4718, 1922 observations					
Liquidity Proxy	EFFCOST	FHT	ROLL	Z	HLS
EFFCOST	1.0000				

FHT	0.7040	1.0000			
	0.0000	-----			
ROLL	0.6496	0.6403	1.0000		
	0.0000	0.0000	-----		
Z	0.2863	0.5860	0.1511	1.0000	
	0.0000	0.0000	0.0000	-----	
HLS	0.6886	0.6829	0.6668	0.2699	1.0000
	0.0000	0.0000	0.0000	0.0000	-----

Panel E: 4718<=VOL<8465, 1922 observations					
Liquidity Proxy	EFFCOST	FHT	ROLL	Z	HLS
EFFCOST	1.0000				

FHT	0.7303	1.0000			
	0.0000	-----			
ROLL	0.6343	0.6686	1.0000		
	0.0000	0.0000	-----		
Z	0.3270	0.5599	0.1611	1.0000	
	0.0000	0.0000	0.0000	-----	
HLS	0.7702	0.7353	0.7402	0.2744	1.0000
	0.0000	0.0000	0.0000	0.0000	-----

Appendix II. Cont'd

Panel F: 8465<=VOL<14907, 1922 observations					
Liquidity Proxy	EFFCOST	FHT	ROLL	Z	HLS
EFFCOST	1.0000				

FHT	0.7922	1.0000			
	0.0000	-----			
ROLL	0.5832	0.6131	1.0000		
	0.0000	0.0000	-----		
Z	0.4222	0.5700	0.1702	1.0000	
	0.0000	0.0000	0.0000	-----	
HLS	0.8008	0.8168	0.7042	0.3206	1.0000
	0.0000	0.0000	0.0000	0.0000	-----

Panel G: 14907<=VOL<27083, 1922 observations					
Liquidity Proxy	EFFCOST	FHT	ROLL	Z	HLS
EFFCOST	1.0000				

FHT	0.7396	1.0000			
	0.0000	-----			
ROLL	0.5401	0.4781	1.0000		
	0.0000	0.0000	-----		
Z	0.3829	0.6310	0.1388	1.0000	
	0.0000	0.0000	0.0000	-----	
HLS	0.8003	0.6629	0.6144	0.2536	1.0000
	0.0000	0.0000	0.0000	0.0000	-----

Panel H: 27083<=VOL<54445, 1922 observations					
Liquidity Proxy	EFFCOST	FHT	ROLL	Z	HLS
EFFCOST	1.0000				

FHT	0.6878	1.0000			
	0.0000	-----			
ROLL	0.4234	0.3773	1.0000		
	0.0000	0.0000	-----		
Z	0.3422	0.6431	0.0857	1.0000	
	0.0000	0.0000	0.0002	-----	
HLS	0.7584	0.6666	0.5489	0.2265	1.0000

	0.0000	0.0000	0.0000	0.0000	-----

Appendix II. Cont'd

Panel I: 54445<=VOL<134987, 1922 observations					
Liquidity Proxy	EFFCOST	FHT	ROLL	Z	HLS
EFFCOST	1.0000				

FHT	0.7912	1.0000			
	0.0000	-----			
ROLL	0.4612	0.4319	1.0000		
	0.0000	0.0000	-----		
Z	0.4339	0.6656	0.1117	1.0000	
	0.0000	0.0000	0.0000	-----	
HLS	0.8031	0.7054	0.5656	0.2347	1.0000
	0.0000	0.0000	0.0000	0.0000	-----

Panel J: 134987<=VOL, 1922 observations					
Liquidity Proxy	EFFCOST	FHT	ROLL	Z	HLS
EFFCOST	1.0000				

FHT	0.6036	1.0000			
	0.0000	-----			
ROLL	0.4556	0.5784	1.0000		
	0.0000	0.0000	-----		
Z	0.4904	0.4033	0.1390	1.0000	
	0.0000	0.0000	0.0000	-----	
HLS	0.6830	0.9292	0.6530	0.2679	1.0000
	0.0000	0.0000	0.0000	0.0000	-----

Table 1. Descriptive statistics of daily returns for each sample period

This table provides descriptive statistics (mean, median, maximum, minimum, std. dev, skewness, kurtosis, Jarque-Bera test and number of observations) for stock returns based on the pooled sample of cross-sectional observations for each sample used in estimating each year of forecasted returns and variances for our VaR model. For example, daily data for the six-year period 1988-1993 are used to obtain the 1993 estimates. The sample includes all companies from the NYSE, Amex, and NASDAQ that were randomly selected by Hasbrouck (2006), have at least 60 months of data in CRSP, and have TAQ-based, trade-weighted proportional (%) effective spreads provided by Hasbrouck (2006) as series effcoslog. The effective spreads are averaged across days within a month. Jarque-Bera values are divided by 1000 for purposes of presentation.

Statistic	**1988-1993**	**1989-1994**	**1990-1995**	**1991-1996**	**1992-1997**	**1993-1998**	**1994-1999**	**1995-2000**	**1996-2001**	**1997-2002**	**1998-2003**
Mean	0.0012	0.0009	0.0010	0.0016	0.0013	0.0011	0.0011	0.0011	0.0012	0.0008	0.0010
Median	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Maximum	2.0000	2.0000	1.1429	2.3333	2.0000	6.5000	2.0476	5.8571	12.6923	3.7000	2.7037
Minimum	-0.7728	-0.7600	-0.7284	-0.7288	-0.5714	-0.7500	-0.6672	-0.7813	-0.8167	-0.8438	-0.7104
Std. Dev.	0.0458	0.0424	0.0411	0.0488	0.0481	0.0513	0.0459	0.0545	0.0613	0.0481	0.0473
Skewness	2.1872	2.4133	1.5806	2.0974	1.7802	10.2171	2.4818	6.6974	34.8807	3.8433	2.8722
Kurtosis	54.6062	70.3433	34.2821	55.5686	39.8639	1007.6700	62.8190	509.8941	6407.9500	179.9512	98.0649
Jarque-Bera	28669	41711	9075	21997	16668	11900000	44280	3130000	512000000	428000	125000
Observations	256516	219608	220322	189829	291649	282422	294961	292461	299508	327615	330795

Table 2. Descriptive statistics of liquidity proxies for each sample period

This table provides descriptive statistics (mean, median, maximum, minimum, std. dev, skewness, kurtosis, Jarque-Bera test and number of observations) for spread estimates based on the pooled sample of monthly time series and cross-sectional observations for each sample used in estimating one year of moving-average standard deviations and empirical percentiles of each proxy. For example, daily data for the six-year period 1989-1993 is used to estimate the 1993 estimates. The sample includes all companies from the NYSE, Amex, and NASDAQ that were randomly selected by Hasbrouck (2006), have at least 60 months of data in CRSP, and have TAQ-based, trade-weighted proportional (%) effective spreads provided by Hasbrouck (2006) as series effcoslog. The effective spreads are averaged across days within a month. FHT is the inverse of the cumulative normal distribution function of Z+1 divided by 2 which is then multiplied by 2 times the standard deviation of returns for that month. Roll is two times the square root of –1 times the autocovariance of daily returns for the month in question. Z is the % of zero-return days within a month. HLS is the equally weighted average of the high-low spread estimator across all overlapping two-day periods within the month. N is the number of observations. Jarque-Bera values are divided by 1000 for purposes of presentation.

	1989-1993				**1989-1994**				**1990-1995**			
Statistic	**FHT**	**ROLL**	**Z**	**HLS**	**FHT**	**ROLL**	**Z**	**HLS**	**FHT**	**ROLL**	**Z**	**HLS**
Mean	0.0311	0.0290	0.3128	0.0182	0.0257	0.0261	0.2842	0.0156	0.0257	0.0261	0.2842	0.0156
Median	0.0180	0.0152	0.2727	0.0099	0.0146	0.0147	0.2500	0.0088	0.0146	0.0147	0.2500	0.0088
Maximum	0.9069	0.5661	1.0000	0.5128	0.3883	0.5285	1.0000	0.4444	0.3883	0.5285	1.0000	0.4444
Minimum	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Std. Dev.	0.0394	0.0413	0.2093	0.0245	0.0319	0.0371	0.1948	0.0214	0.0319	0.0371	0.1948	0.0214
Skewness	4.2270	3.1902	1.1592	4.8070	3.0958	3.3552	1.2430	5.9312	3.0958	3.3552	1.2430	5.9312
Kurtosis	46.0355	20.2367	4.3964	47.2534	17.0982	21.9557	4.8381	74.1140	17.0982	21.9557	4.8381	74.1140
Jarque-Bera	934	166	4	903	100	171	4	2041	100	171	4	2041
N	11656	11802	11802	10569	10075	10145	10145	9426	10075	10145	10145	9426

Table 2. Cont'd

	1991-1996				1992-1997				1993-1998			
Statistic	**FHT**	**ROLL**	**Z**	**HLS**	**FHT**	**ROLL**	**Z**	**HLS**	**FHT**	**ROLL**	**Z**	**HLS**
Mean	0.0327	0.0333	0.3072	0.0206	0.0258	0.0281	0.2568	0.0182	0.0233	0.0275	0.2408	0.0165
Median	0.0201	0.0197	0.2727	0.0124	0.0156	0.0161	0.2273	0.0108	0.0132	0.0170	0.2174	0.0106
Maximum	0.7745	0.4246	1.0000	0.4615	1.2795	0.4042	1.0000	0.3501	1.3784	0.7809	1.0000	0.7916
Minimum	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Std. Dev.	0.0400	0.0425	0.1885	0.0246	0.0333	0.0375	0.1584	0.0216	0.0330	0.0369	0.1581	0.0210
Skewness	4.2668	2.5204	1.1765	3.9525	8.1341	2.6779	1.3279	3.7454	11.0955	4.0497	1.2116	12.0434
Kurtosis	38.4588	12.9334	4.6428	36.2743	221.9482	14.2612	6.0610	30.1320	350.6430	43.9277	5.3372	346.0119
Jarque-Bera	497	47	3	410	19590	63	7	315	44732	642	4	42964
N	8959	9008	9008	8403	9754	9792	9792	9537	8847	8856	8856	8721

	1994-1999				1995-2000				1996-2001			
Statistic	**FHT**	**ROLL**	**Z**	**HLS**	**FHT**	**ROLL**	**Z**	**HLS**	**FHT**	**ROLL**	**Z**	**HLS**
Mean	0.0198	0.0256	0.2200	0.0152	0.0206	0.0281	0.2021	0.0165	0.0202	0.0275	0.1876	0.0152
Median	0.0119	0.0157	0.1905	0.0102	0.0124	0.0179	0.1818	0.0112	0.0106	0.0163	0.1429	0.0104
Maximum	0.3546	0.4959	0.9500	0.3333	0.2949	0.3803	1.0000	0.2963	0.8217	0.6088	1.0000	1.2000
Minimum	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Std. Dev.	0.0245	0.0340	0.1430	0.0167	0.0238	0.0355	0.1432	0.0175	0.0295	0.0373	0.1446	0.0207
Skewness	3.6779	3.0773	0.8743	3.7194	2.8364	2.3579	1.1561	3.7860	6.3776	3.1994	1.0911	21.7948
Kurtosis	25.4602	20.3871	3.8464	31.0797	16.0168	11.7839	4.9186	33.4854	97.1479	24.0609	4.1706	1085.7020
Jarque-Bera	226	138	2	339	81	40	4	394	3899	209	3	504000
N	9720	9720	9720	9634	9645	9648	9648	9579	10367	10368	10368	10295

Table 2. Cont'd

	1997-2002				1998-2003			
Statistic	**FHT**	**ROLL**	**Z**	**HLS**	**FHT**	**ROLL**	**Z**	**HLS**
Mean	0.0131	0.0223	0.1442	0.0132	0.0122	0.0231	0.1213	0.0132
Median	0.0076	0.0151	0.0952	0.0103	0.0065	0.0154	0.0909	0.0101
Maximum	0.3017	1.1535	1.0000	0.1363	0.2359	0.3968	1.0000	0.2222
Minimum	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Std. Dev.	0.0169	0.0300	0.1249	0.0109	0.0171	0.0299	0.1139	0.0115
Skewness	4.1787	6.9292	1.5099	2.4573	3.9541	2.9236	1.8880	3.6262
Kurtosis	37.4098	185.6631	6.2265	14.2299	28.2663	20.3189	8.5690	33.3106
Jarque-Bera	647	17316	10	77	368	175	24	508
Observations	12381	12384	12384	12319	12598	12600	12600	12541

Table 3. Descriptive statistics for the liquidity proxies and the "true" effective spreads

This table provides descriptive statistics (mean, median, maximum, minimum, std. dev, skewness, kurtosis, Jarque-Bera test and number of observations) for the various spread estimates based on the pooled sample of monthly time series and cross-sectional observations from 1993 to 2003. The sample includes all companies from the NYSE, Amex, and NASDAQ that were randomly selected by Hasbrouck (2006), have at least 60 months of data in CRSP, and have TAQ-based, trade-weighted proportional (%) effective spreads provided by Hasbrouck (2006) as series effcoslog. The effective spreads are averaged across days within a month. FHT is the inverse of the cumulative normal distribution function of Z+1 divided by 2 which is then multiplied by 2 times the standard deviation of returns for that month. Roll is two times the square root of –1 times the autocovariance of daily returns for the month in question. Z is the % of zero-return days within a month. HLS is the equally weighted average of the high-low spread estimates across all overlapping two-day periods within the month. Jarque-Bera values are divided by 1000 for purposes of presentation.

	1993-2003				
Statistic	**EFFCOST**	**FHT**	**ROLL**	**Z**	**HLS**
Mean	0.0254	0.0182	0.0260	0.1835	0.0150
Median	0.0124	0.0092	0.0158	0.1429	0.0098
Maximum	0.5918	1.3784	1.1535	1.0000	0.7916
Minimum	0.0003	0.0000	0.0000	0.0000	0.0000
Std. Dev.	0.0365	0.0278	0.0362	0.1504	0.0184
Skewness	3.9889	9.9656	4.5614	1.2667	8.6387
Kurtosis	28.7134	320.0143	64.9360	5.0788	209.1238
Jarque-Bera	566	102000	3958	11	42780
Observations	18735	24222	24240	24240	23997

Table 4. Cross sectional correlation matrix of liquidity proxy estimates and "true" effective spreads

This table provides the correlations (and p-values) between the monthly estimates for the four liquidity proxies with the "true" effective spreads for the 1993-2003 period. The sample includes all companies from the NYSE, Amex, and NASDAQ that were randomly selected by Hasbrouck (2006), have at least 60 months of data in CRSP, and have TAQ-based, trade-weighted proportional (%) effective spreads provided by Hasbrouck (2006) as series effcoslog. The effective spreads are averaged across days within a month. FHT is the inverse of the cumulative normal distribution function of Z+1 divided by 2 which is then multiplied by 2 times the standard deviation of returns for that month. Roll is two times the square root of –1 times the autocovariance of daily returns for the month in question. Z is the % of zero-return days within a month. HLS is the equally weighted average of the high-low spread estimator across all overlapping two-day periods within the month. The number of observations is 18, 589.

Liquidity Proxy	EFFCOST	FHT	ROLL	Z	HLS
EFFCOST	1.0000				

FHT	0.7303	1.0000			
	0.0000	-----			
ROLL	0.6876	0.6434	1.0000		
	0.0000	0.0000	-----		
Z	0.4482	0.5811	0.2184	1.0000	
	0.0000	0.0000	0.0000	-----	
HLS	0.6115	0.6907	0.6206	0.2420	1.0000
	0.0000	0.0000	0.0000	0.0000	-----

Table 5. Back-testing results for VaR model using sum of forecasted variance methodology

Each model is backtested for the years 1993-2003. The sample includes all companies from the NYSE, Amex, and NASDAQ that were randomly selected by Hasbrouck (2006), have at least 60 months of data in CRSP, and have TAQ-based, trade-weighted proportional (%) effective spreads provided by Hasbrouck (2006) as series effcoslog. The effective spreads are averaged across days within a month. Unconditional coverage (LRunc), independence (LRind) and conditional coverage (LRcond) test results as well as % of violations (number of violations/total observations) are reported for the monthly unadjusted VaR (U-VaR) model against unadjusted returns and the liquidity-adjusted VaR (Z L-VaR, Roll L-VaR, and FHT L-VaR) models against liquidity cost adjusted returns for each year. Liquidity-adjusted returns are calculated as log returns minus log effective cost (effcostlog). VaR are the expected monthly loss for a confidence level of 95% where the monthly return variance is calculated by summing the forecasted daily variance over 22 days from a GARCH(1,1) model. L-VaR are VaR models adjusted for liquidity costs by adding the expected average monthly liquidity cost as estimated using each of the four proxies for a confidence level of 95% using a 60 month rolling mean, standard deviation and empirical percentile. Z is the % of zero-return days within a month. Roll is two times the square root of –1 times the autocovariance of daily returns for the month in question. FHT is the inverse of the cumulative normal distribution function of Z+1 divided by 2 which is then multiplied by 2 times the standard deviation of returns for that month. HLS is the equally weighted average of the high-low spread estimates across all overlapping two-day periods within the month.

		1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	TOTAL
U-VaR	**% Violations**	**2.57%**	**3.64%**	**2.27%**	**2.78%**	**5.15%**	**8.60%**	**7.04%**	**8.08%**	**4.98%**	**5.38%**	**1.76%**	**4.75%**
	LRunc	30.34	7.73	33.43	18.61	0.07	33.52	12.63	27.36	0.00	0.61	61.11	
	LRind	0.48	0.02	1.73	2.20	10.51	5.27	8.48	4.73	0.52	11.42	2.64	
	LRcond	30.82	7.75	35.16	20.81	10.59	38.79	21.11	32.10	0.52	12.02	63.75	
Z L-VaR	**% Violations**	**0.16%**	**0.34%**	**0.18%**	**0.07%**	**0.50%**	**2.37%**	**0.69%**	**1.95%**	**1.13%**	**1.67%**	**0.63%**	**0.88%**
	LRunc	167.48	137.26	143.18	137.27	107.10	33.61	96.90	40.34	76.69	63.24	130.53	
	LRind	0.01	0.04	0.01	0.00	0.09	1.27	0.15	0.27	0.47	2.31	3.81	
	LRcond	167.49	137.31	143.19	137.27	107.19	34.88	97.05	40.61	77.16	65.55	134.34	
Roll L-VarR	**% Violations**	**2.11%**	**2.98%**	**1.88%**	**1.94%**	**4.03%**	**7.51%**	**5.77%**	**6.65%**	**3.68%**	**4.23%**	**1.50%**	**3.84%**
	LRunc	42.22	17.65	44.05	36.64	3.30	16.53	1.91	8.35	6.84	2.69	72.39	
	LRind	1.53	0.25	1.14	1.09	7.37	2.22	12.15	2.14	4.68	11.00	3.86	
	LRcond	43.75	17.90	45.19	37.72	10.68	18.75	14.06	10.49	11.52	13.69	76.25	
FHT L-VaR	**% Violations**	**2.27%**	**3.21%**	**2.06%**	**2.22%**	**4.29%**	**8.01%**	**6.21%**	**7.53%**	**4.39%**	**4.97%**	**1.75%**	**4.26%**
	LRunc	37.09	13.66	38.27	29.37	1.76	23.09	4.58	18.75	1.39	0.01	60.78	
	LRind	1.12	0.08	1.34	1.40	8.46	2.16	9.25	1.99	2.31	12.72	2.79	
	LRcond	38.21	13.74	39.61	30.77	10.22	25.25	13.83	20.74	3.70	12.72	63.57	
HLS L-VaR	**% Violations**	**2.49%**	**3.60%**	**2.43%**	**2.64%**	**4.69%**	**8.43%**	**6.78%**	**7.91%**	**4.45%**	**4.45%**	**1.75%**	**4.51%**
	LRunc	29.64	8.04	27.91	20.37	0.32	29.46	9.57	24.31	1.13	0.11	60.78	
	LRind	0.78	0.00	1.92	1.80	6.70	1.24	14.79	4.20	2.45	11.05	60.78	
	LRcond	30.42	8.04	29.83	22.17	7.03	30.70	24.36	28.51	3.58	11.16	121.57	

Table 6. Back-testing results for VaR models using the square root of time methodology

This table provides back test results for various L-VaR models for the years 1993-2003. The sample includes all companies from the NYSE, Amex, and NASDAQ that were randomly selected by Hasbrouck (2006), have at least 60 months of data in CRSP, and have TAQ-based, trade-weighted proportional (%) effective spreads provided by Hasbrouck (2006) as series effcoslog. The effective spreads are averaged across days within a month. Unconditional coverage (LRunc), independence (LRind) and conditional coverage (LRcond) test results as well as % of violations (number of violations/total observations) for the monthly unadjusted VaR (U-VaR) model against unadjusted returns and the liquidity adjusted VaR (Z L-VaR, Roll L-VaR, and FHT L-VaR) models against liquidity cost adjusted returns for each year are presented. Liquidity-adjusted returns are calculated as log returns minus log effective cost (effcostlog). VaR are the expected monthly loss for a confidence level of 95%. The monthly return variance is calculated by multiplying the forecasted one-day ahead standard deviation by the square root of the number of days in a month (22) from a GARCH(1,1) model. The L-VaR are VaR models adjusted for liquidity cost by adding the expected average monthly liquidity cost as estimated by the Roll, HLS, Z and FHT proxies for a confidence level of 95% using a 60 month rolling mean, standard deviation and empirical percentile. Z is the % of zero-return days within a month. Roll is two times the square root of –1 times the autocovariance of daily returns for the month in question. FHT is the inverse of the cumulative normal distribution function of Z+1 divided by 2 which is then multiplied by 2 times the standard deviation of returns for that month. HLS is the equally weighted average of the high-low spread estimates across all overlapping two-day periods within the month.

		1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	TOTAL
U-VaR	**% Violations**	**3.51%**	**4.91%**	**3.26%**	**3.57%**	**5.51%**	**8.88%**	**7.53%**	**8.58%**	**5.90%**	**6.25%**	**2.29%**	**5.47%**
	LRunc	10.28	0.03	12.35	7.18	0.88	38.31	19.04	36.11	2.81	6.31	40.47	
	LRind	0.26	0.30	0.27	0.72	6.14	0.75	9.76	1.65	0.25	7.84	3.55	
	LRcond	10.54	0.33	12.62	7.91	7.02	39.06	28.80	37.75	3.06	14.15	44.02	
Z L-VaR	**% Violations**	**0.16%**	**0.45%**	**0.24%**	**0.07%**	**0.64%**	**2.04%**	**0.82%**	**2.13%**	**1.36%**	**1.82%**	**0.82%**	**0.96%**
	LRunc	167.48	126.90	136.75	137.27	98.26	33.61	89.15	34.78	65.24	56.72	114.73	
	LRind	0.01	0.07	0.02	0.00	0.14	1.27	0.22	1.43	0.70	4.34	3.25	
	LRcond	167.49	126.97	136.77	137.27	98.40	34.88	89.36	36.21	65.94	61.06	117.98	
Roll L-VarR	**% Violations**	**3.11%**	**3.94%**	**2.85%**	**2.08%**	**4.54%**	**8.08%**	**6.09%**	**6.91%**	**4.68%**	**5.51%**	**1.79%**	**4.51%**
	LRunc	16.35	4.50	18.95	32.87	0.71	24.10	3.71	10.94	0.36	1.06	58.64	
	LRind	0.06	0.08	0.03	1.22	9.66	0.15	12.73	2.41	2.97	14.08	2.70	
	LRcond	16.41	4.58	18.98	34.09	10.37	24.25	16.44	13.34	3.33	15.14	61.33	
FHT L-VaR	**% Violations**	**3.16%**	**4.22%**	**2.91%**	**2.29%**	**4.86%**	**8.43%**	**6.84%**	**7.78%**	**5.51%**	**6.15%**	**2.18%**	**4.94%**
	LRunc	15.38	2.38	17.80	27.72	0.06	29.46	10.22	22.39	0.90	5.25	43.35	
	LRind	0.04	0.25	0.05	1.35	5.41	0.00	14.54	1.10	0.33	9.05	4.06	
	LRcond	15.42	2.63	17.85	29.07	5.47	29.46	24.77	23.48	1.23	14.30	47.41	
HLS L-VaR	**% Violations**	**3.52%**	**4.50%**	**3.04%**	**2.91%**	**5.07%**	**8.78%**	**7.28%**	**7.97%**	**5.69%**	**6.29%**	**2.08%**	**5.20%**
	LRunc	9.38	0.95	15.35	15.46	0.02	35.27	15.35	25.30	1.62	6.64	46.88	
	LRind	0.40	0.01	0.12	0.01	6.85	0.25	11.12	1.58	0.06	8.03	1.70	
	LRcond	9.78	0.96	15.47	15.47	6.87	35.53	26.47	26.88	1.68	14.67	48.58	

Table 7. Back-testing results for liquidity adjustment

Each model is backtested for the years 1993-2003. The sample includes all companies from the NYSE, Amex, and NASDAQ that were randomly selected by Hasbrouck (2006), have at least 60 months of data in CRSP, and have TAQ-based, trade-weighted proportional (%) effective spreads provided by Hasbrouck (2006) as series effcoslog. The effective spreads are averaged across days within a month. Results show cross sectional Unconditional coverage (LRunc), independence (LRind) and conditional coverage (LRcond) test results as well as % of violations (number of violations/total observations) for the monthly worst expected liquidity cost as estimated from liquidity proxies against true effective cost (effcostlog). The worst expected liquidity cost are the expected monthly liquidity cost (as estimated by the Roll, HLS, Z and FHT proxies) for a confidence level of 95% using a 60-month rolling mean, standard deviation and empirical percentile. Z is the % of 0 return days within a month. Roll is two times the square root of –1 times the autocovariance of daily returns for the month in question. FHT is the inverse of the cumulative normal distribution function of Z+1 divided by 2 which is then multiplied by 2 times the standard deviation of returns for that month. HLS is the equally weighted average of the high-low spread estimator across all overlapping 2-day periods within the month.

		1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	TOTAL
Z	% Violations	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.06%**	**0.06%**	**0.00%**	**0.00%**	**0.00%**	**0.01%**
	LRunc	194.50	182.19	169.37	147.93	160.34	146.08	152.66	152.56	173.06	208.66	211.64	
	LRind	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
	LRcond	194.50	182.19	169.37	147.93	160.34	146.08	152.67	152.56	173.06	208.66	211.64	
Roll	% Violations	**7.70%**	**4.73%**	**5.45%**	**4.02%**	**1.79%**	**3.09%**	**2.82%**	**2.70%**	**2.19%**	**2.70%**	**1.55%**	**3.52%**
	LRunc	25.17	0.28	0.69	3.10	44.50	12.59	18.74	21.17	35.11	26.90	69.94	
	LRind	230.96	179.51	177.85	87.70	77.67	128.11	39.31	27.57	91.92	131.72	71.41	
	LRcond	256.12	179.79	178.54	90.80	122.17	140.70	58.05	48.74	127.02	158.62	141.35	
FHT	% Violations	**11.39 %**	**8.78%**	**8.48%**	**7.49%**	**4.75%**	**4.42%**	**6.27%**	**9.04%**	**6.16%**	**7.52%**	**5.56%**	**7.26%**
	LRunc	121.70	44.12	35.14	16.43	0.24	1.03	5.05	44.62	4.50	23.75	1.38	
	LRind	264.66	244.19	178.88	148.53	151.21	54.15	138.28	172.20	122.75	198.24	238.26	
	LRcond	386.36	288.31	214.02	164.96	151.45	55.19	143.34	216.82	127.25	221.98	239.65	
HLS	% Violations	**32.27 %**	**26.91 %**	**27.01 %**	**20.87 %**	**15.16 %**	**13.55 %**	**17.13 %**	**21.72 %**	**18.26 %**	**15.19 %**	**12.89 %**	**20.09 %**
	LRunc	inf	inf	inf	inf	inf	152.63	inf	inf	inf	inf	inf	
	LRind	inf	inf	inf	446.88	385.03	372.76	451.95	inf	552.67	inf	818.12	
	LRcond	inf	inf	inf	inf	inf	525.39	inf	inf	inf	inf	inf	

Table 8. Back-testing result for VaR model sampled by volume traded using sum of forecasted variance methodology

This table provides back test results for various VaR models for each volume traded decile. The sample includes all companies from the NYSE, Amex, and NASDAQ that were randomly selected by Hasbrouck (2006), have at least 60 months of data in CRSP, and have TAQ-based, trade-weighted proportional (%) effective spreads provided by Hasbrouck (2006) as series effcoslog. The effective spreads are averaged across days within a month. Unconditional coverage (LRunc), independence (LRind) and conditional coverage (LRcond) test results as well as % of violations (number of violations/total observations) are reported for the monthly unadjusted VaR (U-VaR) model against unadjusted returns and the liquidity-adjusted VaR (Z L-VaR, Roll L-VaR, and FHT L-VaR) models against liquidity cost adjusted returns for each year. Liquidity-adjusted returns are calculated as log returns minus log effective cost (effcostlog). VaR are the expected monthly loss for a confidence level of 95% where the monthly return variance is calculated by summing the forecasted daily variance over 22 days from a GARCH(1,1) model. L-VaR are VaR models adjusted for liquidity costs by adding the expected average monthly liquidity cost as estimated using each of the four proxies for a confidence level of 95% using a 60 month rolling mean, standard deviation and empirical percentile. Z is the % of zero-return days within a month. Roll is two times the square root of –1 times the autocovariance of daily returns for the month in question. FHT is the inverse of the cumulative normal distribution function of Z+1 divided by 2 which is then multiplied by 2 times the standard deviation of returns for that month. HLS is the equally weighted average of the high-low spread estimates across all overlapping two-day periods within the month.

		1st decile	**2nd decile**	**3rd decile**	**4th decile**	**5th decile**
		VOL<414	**414<=VOL<1163**	**1163<=VOL<2438**	**2438<=VOL<4718**	**4718<=VOL<8465**
U-VaR	**% Violations**	**2.34%**	**3.06%**	**2.96%**	**3.22%**	**4.21%**
	LRunc	38.34	19.61	24.18	18.42	4.23
	LRind	7.61	9.73	4.72	6.57	5.91
	LRcond	45.95	29.34	28.90	24.99	10.14
Z L-VaR	**% Violations**	**0.05%**	**0.27%**	**0.05%**	**0.43%**	**0.43%**
	LRunc	177.82	150.59	180.05	134.12	135.34
	LRind	7.81	0.02	0.01	0.05	0.07
	LRcond	185.63	150.62	180.06	134.16	135.40
Roll L-VarR	**% Violations**	**2.32%**	**2.46%**	**2.41%**	**2.26%**	**3.48%**
	LRunc	40.77	36.40	35.28	39.66	11.64
	LRind	5.24	12.48	8.26	9.34	4.09
	LRcond	46.01	48.88	43.55	49.00	15.73
FHT L-VaR	**% Violations**	**2.21%**	**2.79%**	**2.68%**	**2.74%**	**3.69%**
	LRunc	44.43	27.39	27.84	25.95	8.62
	LRind	5.93	6.75	9.61	6.59	4.91
	LRcond	50.37	34.14	37.45	32.53	13.53
HLS L-VaR	**% Violations**	**2.55%**	**2.98%**	**2.90%**	**2.97%**	**4.18%**
	LRunc	27.53	18.40	20.19	20.71	3.54
	LRind	9.01	10.02	6.35	5.41	3.65
	LRcond	36.54	28.42	26.54	26.12	7.18

Table 8. Cont'd

		6th decile	7th decile	8th decile	9th decile	10th decile	
		8465<=VOL<14907	**14907<=VOL<27083**	**27083<=VOL<54445**	**54445<=VOL<134987**	**134987<=VOL**	**TOTAL**
U-VaR	**% Violations**	**5.03%**	**4.62%**	**6.24%**	**6.92%**	**8.89%**	**4.75%**
	LRunc	0.11	1.40	4.91	13.39	47.87	
	LRind	4.12	0.68	5.30	7.00	3.09	
	LRcond	4.23	2.09	10.22	20.39	50.96	
Z L-VaR	**% Violations**	**0.48%**	**0.73%**	**1.31%**	**1.74%**	**3.30%**	**0.88%**
	LRunc	131.20	119.92	81.82	60.32	14.58	
	LRind	0.07	0.12	7.81	0.77	0.56	
	LRcond	131.27	120.05	89.64	61.10	15.14	
Roll L-VarR	**% Violations**	**3.52%**	**3.52%**	**4.93%**	**5.70%**	**7.83%**	**3.84%**
	LRunc	11.10	11.10	0.11	1.38	25.46	
	LRind	6.59	0.01	1.74	5.80	1.21	
	LRcond	17.69	11.10	1.85	7.19	26.67	
FHT L-VaR	**% Violations**	**3.94%**	**3.79%**	**5.62%**	**6.39%**	**8.74%**	**4.26%**
	LRunc	5.73	7.54	1.04	6.12	43.37	
	LRind	4.53	0.03	5.00	5.69	0.81	
	LRcond	10.26	7.57	6.04	11.80	44.18	
HLS L-VaR	**% Violations**	**4.37%**	**4.00%**	**5.88%**	**6.54%**	**8.69%**	**4.51%**
	LRunc	2.18	5.20	2.34	7.65	42.20	
	LRind	3.24	0.08	6.32	8.59	0.48	
	LRcond	5.42	5.28	8.67	16.24	42.68	

Table 9. Back-testing results for VaR models sampled by volume traded using the square root of time methodology

This table provides back test results for each volume traded decile. The sample includes all companies from the NYSE, Amex, and NASDAQ that were randomly selected by Hasbrouck (2006), have at least 60 months of data in CRSP, and have TAQ-based, trade-weighted proportional (%) effective spreads provided by Hasbrouck (2006) as series effcoslog. The effective spreads are averaged across days within a month. Unconditional coverage (LRunc), independence (LRind) and conditional coverage (LRcond) test results as well as % of violations (number of violations/total observations) for the monthly unadjusted VaR (U-VaR) model against unadjusted returns and the liquidity adjusted VaR (Z L-VaR, Roll L-VaR, and FHT L-VaR) models against liquidity cost adjusted returns for each year are presented. Liquidity-adjusted returns are calculated as log returns minus log effective cost (effcostlog). VaR are the expected monthly loss for a confidence level of 95%. The monthly return variance is calculated by multiplying the forecasted one-day ahead standard deviation by the square root of the number of days in a month (22) from a GARCH(1,1) model. The L-VaR are VaR models adjusted for liquidity cost by adding the expected average monthly liquidity cost as estimated by the Roll, HLS, Z and FHT proxies for a confidence level of 95% using a 60 month rolling mean, standard deviation and empirical percentile. Z is the % of zero-return days within a month. Roll is two times the square root of –1 times the autocovariance of daily returns for the month in question. FHT is the inverse of the cumulative normal distribution function of Z+1 divided by 2 which is then multiplied by 2 times the standard deviation of returns for that month. HLS is the equally weighted average of the high-low spread estimates across all overlapping two-day periods within the month.

		1st decile	2nd decile	3rd decile	4th decile	5th decile
		VOL<414	**414<=VOL <1163**	**1163<=VOL <2438**	**2438<=VOL <4718**	**4718<=VOL <8465**
U-VaR	**% Violations**	**3.56%**	**4.10%**	**3.64%**	**4.11%**	**4.68%**
	LRunc	13.45	4.34	9.62	4.28	0.72
	LRind	6.11	8.36	5.89	2.96	2.55
	LRcond	19.56	12.70	15.51	7.23	3.27
Z L-VaR	**% Violations**	**0.15%**	**0.27%**	**0.11%**	**0.38%**	**0.64%**
	LRunc	177.82	150.59	171.65	139.24	117.05
	LRind	7.61	0.02	0.02	0.04	0.10
	LRcond	185.43	150.62	171.67	139.28	117.15
Roll L-VarR	**% Violations**	**2.87%**	**3.33%**	**3.16%**	**3.12%**	**4.23%**
	LRunc	22.87	13.84	17.08	17.67	3.18
	LRind	6.24	12.24	13.45	7.62	5.15
	LRcond	29.11	26.07	30.53	25.29	8.33
FHT L-VaR	**% Violations**	**3.08%**	**3.81%**	**3.47%**	**3.82%**	**3.79%**
	LRunc	20.49	8.30	13.27	8.24	0.04
	LRind	5.82	6.37	13.45	2.96	1.99
	LRcond	26.31	14.67	26.72	11.19	2.03
HLS L-VaR	**% Violations**	**3.75%**	**4.16%**	**3.70%**	**3.94%**	**4.66%**
	LRunc	8.85	4.54	8.47	5.63	0.77
	LRind	3.37	9.44	8.37	3.31	6.84
	LRcond	12.22	13.98	16.84	8.93	7.61

Table 9. Cont'd

		6th decile	7th decile	8th decile	9th decile	10th decile	
		8465<=VOL <14907	**14907<=VOL <27083**	**27083<=VOL <54445**	**54445<=VOL <134987**	**134987 <=VOL**	**TOTAL**
U-VaR	**% Violations**	**5.87%**	**5.35%**	**6.71%**	**7.38%**	**9.25%**	**5.47%**
	LRunc	2.29	0.26	9.54	18.61	56.65	
	LRind	0.80	0.38	2.63	5.40	2.11	
	LRcond	3.09	0.63	12.17	24.02	58.76	
Z L-VaR	**% Violations**	**0.69%**	**0.84%**	**1.31%**	**1.74%**	**3.47%**	**0.96%**
	LRunc	113.65	104.24	76.16	55.85	10.35	
	LRind	0.10	0.16	3.30	0.95	0.03	
	LRcond	113.75	104.40	79.46	56.79	10.38	
Roll L-VarR	**% Violations**	**4.37%**	**4.05%**	**5.35%**	**6.23%**	**8.37%**	**4.51%**
	LRunc	2.21	4.72	0.26	4.74	35.50	
	LRind	1.68	0.08	3.21	5.17	0.13	
	LRcond	3.90	4.79	3.46	9.91	35.63	
FHT L-VaR	**% Violations**	**4.79%**	**4.84%**	**5.88%**	**6.70%**	**9.22%**	**4.94%**
	LRunc	0.68	0.54	2.33	9.35	54.45	
	LRind	0.93	0.07	1.58	2.97	0.64	
	LRcond	1.61	0.62	3.91	12.32	55.09	
HLS L-VaR	**% Violations**	**4.91%**	**4.84%**	**6.15%**	**6.70%**	**9.22%**	**5.20%**
	LRunc	0.15	0.27	4.12	9.35	54.45	
	LRind	1.82	0.01	3.15	4.14	1.16	
	LRcond	1.97	0.27	7.27	13.49	55.61	

Table 10. Back-testing results for liquidity adjustment by volume traded

Each model is backtested for each volume traded decile. The sample includes all companies from the NYSE, Amex, and NASDAQ that were randomly selected by Hasbrouck (2006), have at least 60 months of data in CRSP, and have TAQ-based, trade-weighted proportional (%) effective spreads provided by Hasbrouck (2006) as series effcoslog. The effective spreads are averaged across days within a month. Results show cross sectional Unconditional coverage (LRunc), independence (LRind) and conditional coverage (LRcond) test results as well as % of violations (number of violations/total observations) for the monthly worst expected liquidity cost as estimated from liquidity proxies against true effective cost (effcostlog). The worst expected liquidity cost are the expected monthly liquidity cost (as estimated by the Roll, HLS, Z and FHT proxies) for a confidence level of 95% using a 60-month rolling mean, standard deviation and empirical percentile. Z is the % of 0 return days within a month. Roll is two times the square root of –1 times the autocovariance of daily returns for the month in question. FHT is the inverse of the cumulative normal distribution function of Z+1 divided by 2 which is then multiplied by 2 times the standard deviation of returns for that month. HLS is the equally weighted average of the high–low spread estimator across all overlapping 2-day periods within the month.

		1st decile	**2nd decile**	**3rd decile**	**4th decile**	**5th decile**
		VOL<414	**414<=VOL <1163**	**1163<=VOL <2438**	**2438<=VOL <4718**	**4718<=VOL <8465**
Z	**% Violations**	**0.00%**	**0.00%**	**0.00%**	**0.05%**	**0.05%**
	LRunc	inf	inf	inf	178.93	180.25
	LRind	inf	inf	inf	0.00	0.01
	LRcond	inf	inf	inf	178.94	180.26
Roll	**% Violations**	**19.06%**	**6.15%**	**3.12%**	**2.33%**	**1.51%**
	LRunc	inf	5.70	14.17	31.58	60.76
	LRind	inf	138.64	65.36	61.05	44.09
	LRcond	inf	144.34	79.53	92.63	104.85
FHT	**% Violations**	**24.14%**	**15.80%**	**12.02%**	**7.35%**	**4.99%**
	LRunc	inf	inf	inf	20.44	0.03
	LRind	inf	inf	inf	138.90	145.73
	LRcond	inf	inf	inf	159.34	145.76
HLS	**% Violations**	**54.91%**	**48.05%**	**36.12%**	**23.24%**	**16.22%**
	LRunc	inf	inf	inf	inf	inf
	LRind	inf	inf	inf	inf	inf
	LRcond	inf	inf	inf	inf	inf

Table 10. Cont'd

		6th decile	7th decile	8th decile	9th decile	10th decile	
		8465<=VOL <14907	**14907<=VOL <27083**	**27083<=VOL <54445**	**54445<=VOL <134987**	**134987<=VOL**	**TOTAL**
Z	**% Violations**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.01%**
	LRunc	192.25	194.71	195.22	194.20	192.35	
	LRind	0.00	0.00	0.00	0.00	0.00	
	LRcond	192.25	194.71	195.22	194.20	192.35	
Roll	**% Violations**	**1.07%**	**0.79%**	**0.53%**	**0.48%**	**0.21%**	**3.52%**
	LRunc	88.64	107.95	129.19	132.96	158.71	
	LRind	22.62	10.72	0.11	39.20	19.95	
	LRcond	111.26	118.67	129.31	172.17	178.66	
FHT	**% Violations**	**2.45%**	**2.05%**	**1.73%**	**1.48%**	**0.53%**	**7.26%**
	LRunc	31.21	44.15	56.52	67.53	126.62	
	LRind	22.05	40.56	6.19	38.92	40.72	
	LRcond	53.26	84.71	62.71	106.45	167.34	
HLS	**% Violations**	**10.26%**	**5.65%**	**3.63%**	**2.17%**	**0.64%**	**20.09%**
	LRunc	87.76	2.12	7.02	40.28	117.86	
	LRind	314.32	148.44	109.45	124.91	56.86	
	LRcond	402.08	150.56	116.47	165.19	174.72	

Table 11. Descriptive statistics of liquidity proxies and "true" effective spreads during the 2001 crisis

The table provides descriptive statistics (mean, median, maximum, minimum, std. dev., skewness, kurtosis, Jarque-Bera test and number of observations) for monthly spread estimates based on the pooled sample of monthly time series and cross-sectional observations from March to November 2001. The sample includes all companies from the NYSE, Amex, and NASDAQ that were randomly selected by Hasbrouck (2006), have at least 60 months of data in CRSP, and have TAQ-based, trade-weighted proportional (%) effective spreads provided by Hasbrouck (2006) as series effcoslog. The effective spreads are averaged across days within a month. FHT is the inverse of the cumulative normal distribution function of Z+1 divided by 2 which is then multiplied by 2 times the standard deviation of returns for that month. Roll is two times the square root of –1 times the autocovariance of daily returns for the month in question. Z is the % of zero-return days within a month. HLS is the equally weighted average of the high-low spread estimator across all overlapping two-day periods within the month. Jarque-Bera values are divided by 1000 for purposes of presentation.

Statistic	EFFCOST	FHT	ROLL	Z	HLS
Mean	0.0208	0.0129	0.0246	0.1306	0.0121
Median	0.0082	0.0055	0.0147	0.0870	0.0091
Maximum	0.3259	0.1722	0.5091	0.7619	0.1438
Minimum	0.0008	0.0000	0.0000	0.0000	0.0000
Std. Dev.	0.0317	0.0189	0.0347	0.1348	0.0111
Skewness	3.9153	3.1087	3.9210	1.3944	3.6330
Kurtosis	26.3161	16.4532	35.1607	4.9023	30.1122
Jarque-Bera	32	15	77	1	55
Observations	1269	1683	1683	1683	1665

Table 12. Cross sectional correlation matrix of liquidity proxies and “true” effective spreads during the 2001 crisis

This table reports the cross sectional correlations (and respective p-values) between the monthly spread measures and the “true” effective spreads from TAQ (EFFCOST) for the period of March to November 2001. The sample includes all companies from the NYSE, Amex, and NASDAQ that were randomly selected by Hasbrouck (2006), have at least 60 months of data in CRSP, and have TAQ-based, trade-weighted proportional (%) effective spreads provided by Hasbrouck (2006) as series effcoslog. The effective spreads are averaged across days within a month. FHT is the inverse of the cumulative normal distribution function of Z+1 divided by 2 which is then multiplied by 2 times the standard deviation of returns for that month. Roll is two times the square root of –1 times the autocovariance of daily returns for the month in question. Z is the % of 0 return days within a month. HLS is the equally weighted average of the high-low spread estimator across all overlapping 2-day periods within the month. The number of observations is 1,265.

Liquidity Proxy	EFFCOST	FHT	ROLL	Z	HLS
EFFCOST	1.0000				

FHT	0.7054	1.0000			
	0.0000	-----			
ROLL	0.6439	0.5494	1.0000		
	0.0000	0.0000	-----		
Z	0.3980	0.6497	0.1659	1.0000	
	0.0000	0.0000	0.0000	-----	
HLS	0.4677	0.4487	0.5773	0.0496	1.0000
	0.0000	0.0000	0.0000	0.0781	-----

Table 13. Back-testing results for VaR model during the 2001 crisis

Each model is backtested from March to November 2001. The sample includes all companies from the NYSE, Amex, and NASDAQ that were randomly selected by Hasbrouck (2006), have at least 60 months of data in CRSP, and have TAQ-based, trade-weighted proportional (%) effective spreads provided by Hasbrouck (2006) as series effcoslog. The effective spreads are averaged across days within a month. Results show cross sectional Unconditional coverage (LRunc), independence (LRind) and conditional coverage (LRcond) test results as well as % of violations (number of violations/total observations) for the monthly unadjusted VaR (U-VaR) model against unadjusted returns and the liquidity adjusted VaR (Z L-VaR, Roll L-VaR, and FHT L-VaR) models against liquidity cost adjusted returns for each year. Liquidity-adjusted returns are calculated as log returns minus log effective cost (effcostlog). VaR are the expected monthly loss for a confidence level of 95% where the monthly return variance is calculated by multiplying the forecasted one-day ahead standard deviation by the square root of the number of days in a month (22 days) and by summing the forecasted daily variance (over 22 days) from a GARCH(1,1) model. L-VaR are VaR models adjusted for liquidity cost by adding the expected average monthly liquidity cost (as estimated by the Roll, HLS, Z and FHT proxies) for a confidence level of 95% using a 60 month rolling mean, standard deviation and empirical percentile. Z is the % of 0 return days within a month. Roll is two times the square root of –1 times the autocovariance of daily returns for the month in question. FHT is the inverse of the cumulative normal distribution function of Z+1 divided by 2 which is then multiplied by 2 times the standard deviation of returns for that month. HLS is the equally weighted average of the high–low spread estimator across all overlapping 2-day periods within the month.

	U-VaR		**Z L-VaR**		**Roll L-VarR**		**FHT L-VaR**		**HLS L-VaR**	
	Σ Var.	**$Time^{0.5}$**	**Σ Var.**	**$Time^{0.5}$**	**Σ Var.**	**$Time^{0.5}$**	**Σ Var.**	**$Time^{0.5}$**	**Σ Var.**	**$Time^{0.5}$**
% Violations	5.63%	6.71%	1.42%	1.73%	4.26%	5.36%	5.12%	6.38%	5.20%	6.62%
LRunc	1.0512	7.2636	47.2367	37.7039	1.5569	0.3359	0.0396	4.7166	0.1065	6.3872
LRind	0.5146	0.2739	0.5488	0.7431	4.8175	3.1129	2.4481	0.4389	2.6077	0.1252
LRcond	1.5658	7.5375	47.7854	38.4470	6.3744	3.4489	2.4877	5.1555	2.7142	6.5124

Table 14. Back-testing results for liquidity adjustment during the 2001 crisis

Each model is backtested from March to November 2001. The sample includes all companies from the NYSE, Amex, and NASDAQ that were randomly selected by Hasbrouck (2006), have at least 60 months of data in CRSP, and have TAQ-based, trade-weighted proportional (%) effective spreads provided by Hasbrouck (2006) as series effcoslog. The effective spreads are averaged across days within a month. Results show cross sectional Unconditional coverage (LRunc), independence (LRind) and conditional coverage (LRcond) test results as well as % of violations (number of violations/total observations) for the monthly worst expected liquidity cost as estimated from liquidity proxies against true effective cost (effcostlog). The worst expected liquidity cost are the expected monthly liquidity cost (as estimated by the Roll, HLS, Z and FHT proxies) for a confidence level of 95% using a 60 month rolling mean, standard deviation and empirical percentile. Z is the % of zero-return days within a month. Roll is two times the square root of –1 times the autocovariance of daily returns for the month in question. FHT is the inverse of the cumulative normal distribution function of Z+1 divided by 2 which is then multiplied by 2 times the standard deviation of returns for that month. HLS is the equally weighted average of the high–low spread estimator across all overlapping 2-day periods within the month

	Z	Roll	FHT	HLS
% Violations	0.00	2.44	6.15	17.65
LRunc	130.1824	21.3573	3.2836	266.4037
LRind	0.0000	89.1257	97.6789	442.0241
LRcond	130.1824	110.4830	100.9624	708.4279

Figure 1. % Violations for VaR model using sum of forecasted variance methodology

Each model is backtested for the years 1993-2003. The sample includes all companies from the NYSE, Amex, and NASDAQ that were randomly selected by Hasbrouck (2006), have at least 60 months of data in CRSP, and have TAQ-based, trade-weighted proportional (%) effective spreads provided by Hasbrouck (2006) as series effcoslog. The effective spreads are averaged across days within a month. % of violations (number of violations/total observations) are reported for the monthly unadjusted VaR (U-VaR) model against unadjusted returns and the liquidity-adjusted VaR (Z L-VaR, Roll L-VaR, and FHT L-VaR) models against liquidity cost adjusted returns for each year. Liquidity-adjusted returns are calculated as log returns minus log effective cost (effcostlog). VaR are the expected monthly loss for a confidence level of 95% where the monthly return variance is calculated by summing the forecasted daily variance over 22 days from a GARCH(1,1) model. L-VaR are VaR models adjusted for liquidity costs by adding the expected average monthly liquidity cost as estimated using each of the four proxies for a confidence level of 95% using a 60 month rolling mean, standard deviation and empirical percentile. Z is the % of zero-return days within a month. Roll is two times the square root of –1 times the autocovariance of daily returns for the month in question. FHT is the inverse of the cumulative normal distribution function of Z+1 divided by 2 which is then multiplied by 2 times the standard deviation of returns for that month. HLS is the equally weighted average of the high-low spread estimates across all overlapping two-day periods within the month.

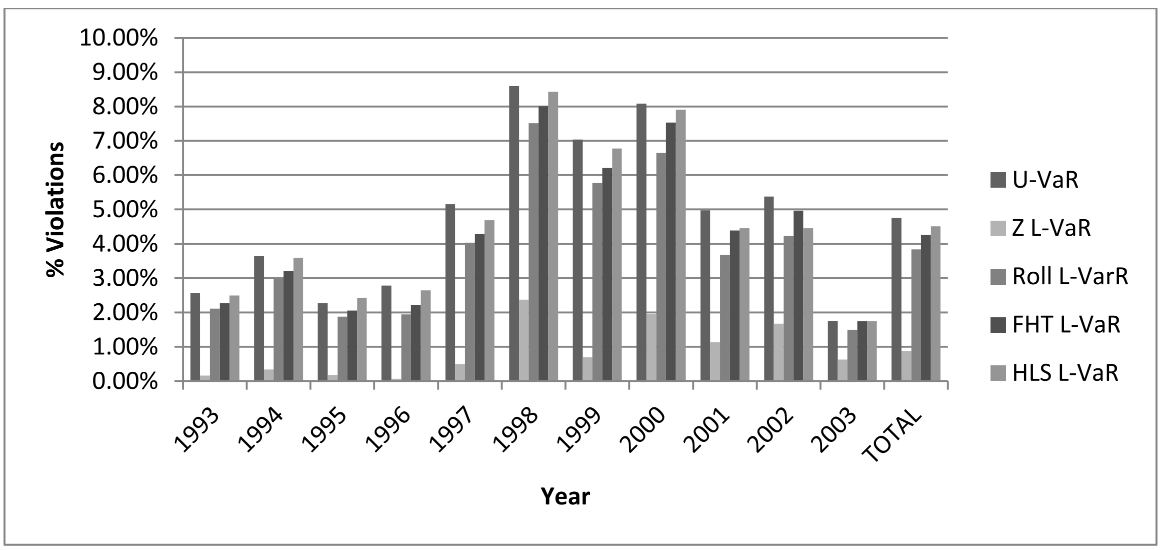

Figure 2. % Violations for VaR model using square root of time methodology

Each model is backtested for the years 1993-2003. The sample includes all companies from the NYSE, Amex, and NASDAQ that were randomly selected by Hasbrouck (2006), have at least 60 months of data in CRSP, and have TAQ-based, trade-weighted proportional (%) effective spreads provided by Hasbrouck (2006) as series effcoslog. The effective spreads are averaged across days within a month. % of violations (number of violations/total observations) are reported for the monthly unadjusted VaR (U-VaR) model against unadjusted returns and the liquidity-adjusted VaR (Z L-VaR, Roll L-VaR, and FHT L-VaR) models against liquidity cost adjusted returns for each year. Liquidity-adjusted returns are calculated as log returns minus log effective cost (effcostlog). VaR are the expected monthly loss for a confidence level of 95% where the monthly return variance is calculated by multiplying the forecasted one-day ahead standard deviation by the square root of the number of days in a month (22) from a GARCH(1,1) model. L-VaR are VaR models adjusted for liquidity costs by adding the expected average monthly liquidity cost as estimated using each of the four proxies for a confidence level of 95% using a 60 month rolling mean, standard deviation and empirical percentile. Z is the % of zero-return days within a month. Roll is two times the square root of –1 times the autocovariance of daily returns for the month in question. FHT is the inverse of the cumulative normal distribution function of Z+1 divided by 2 which is then multiplied by 2 times the standard deviation of returns for that month. HLS is the equally weighted average of the high-low spread estimates across all overlapping two-day periods within the month.

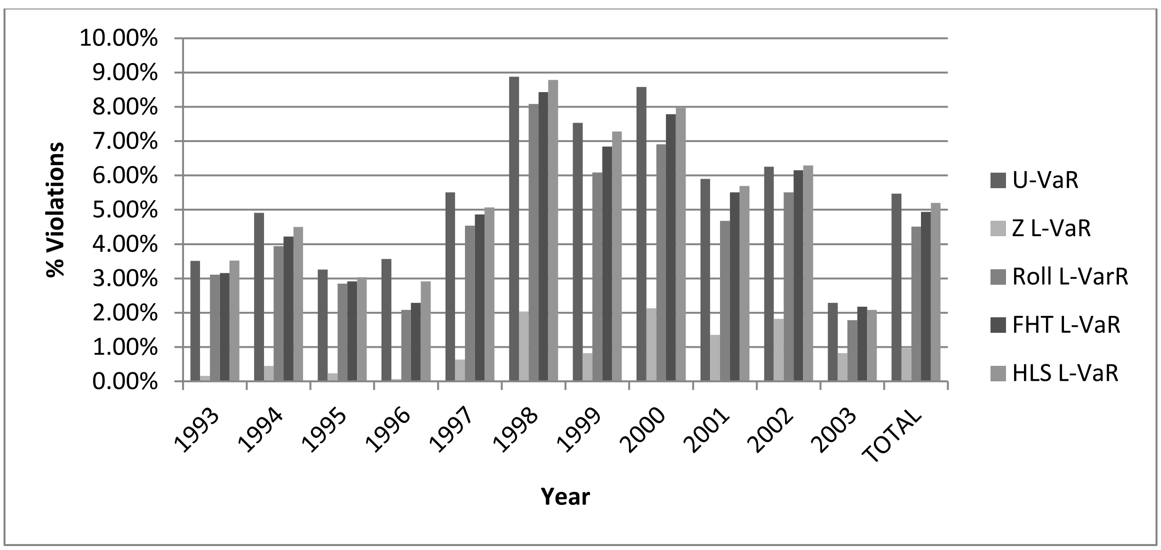

Figure 3. % Violations for liquidity adjustment

Each model is backtested for the years 1993-2003. The sample includes all companies from the NYSE, Amex, and NASDAQ that were randomly selected by Hasbrouck (2006), have at least 60 months of data in CRSP, and have TAQ-based, trade-weighted proportional (%) effective spreads provided by Hasbrouck (2006) as series effcoslog. The effective spreads are averaged across days within a month. Results show cross sectional % of violations (number of violations/total observations) for the monthly worst expected liquidity cost as estimated from liquidity proxies against true effective cost (effcostlog). The worst expected liquidity cost are the expected monthly liquidity cost (as estimated by the Roll, HLS, Z and FHT proxies) for a confidence level of 95% using a 60-month rolling mean, standard deviation and empirical percentile. Z is the % of 0 return days within a month. Roll is two times the square root of –1 times the autocovariance of daily returns for the month in question. FHT is the inverse of the cumulative normal distribution function of Z+1 divided by 2 which is then multiplied by 2 times the standard deviation of returns for that month. HLS is the equally weighted average of the high–low spread estimator across all overlapping 2-day periods within the month.

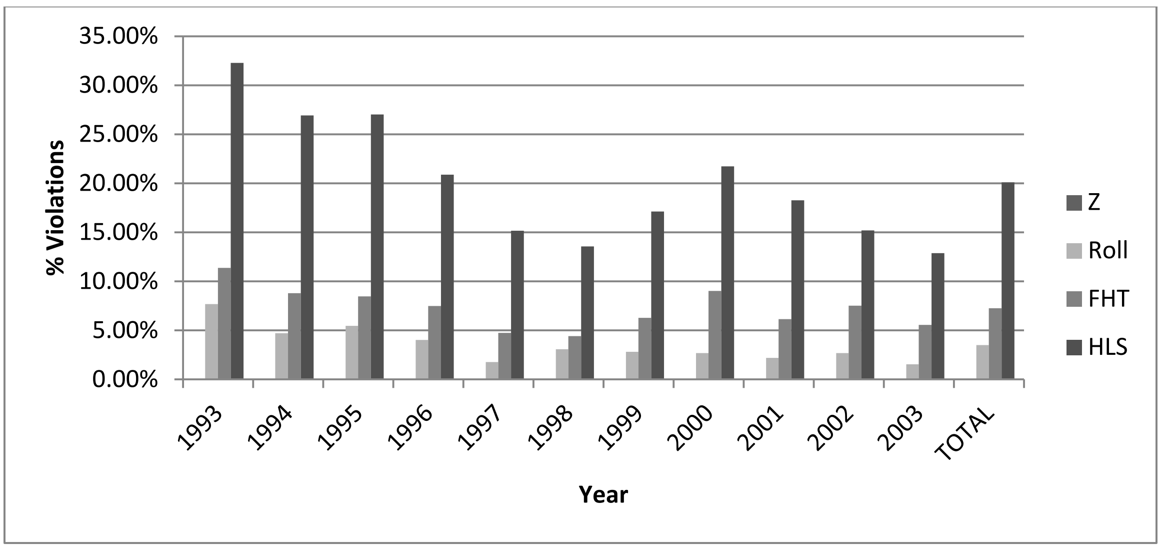

Figure 4. % Violations for VaR model sampled by volume traded using sum of forecasted variance methodology

Each model is backtested for various VaR models for each volume traded (VOL) decile. The sample includes all companies from the NYSE, Amex, and NASDAQ that were randomly selected by Hasbrouck (2006), have at least 60 months of data in CRSP, and have TAQ-based, trade-weighted proportional (%) effective spreads provided by Hasbrouck (2006) as series effcoslog. The effective spreads are averaged across days within a month. % of violations (number of violations/total observations) are reported for the monthly unadjusted VaR (U-VaR) model against unadjusted returns and the liquidity-adjusted VaR (Z L-VaR, Roll L-VaR, and FHT L-VaR) models against liquidity cost adjusted returns for each year. Liquidity-adjusted returns are calculated as log returns minus log effective cost (effcostlog). VaR are the expected monthly loss for a confidence level of 95% where the monthly return variance is calculated by summing the forecasted daily variance over 22 days from a GARCH(1,1) model. L-VaR are VaR models adjusted for liquidity costs by adding the expected average monthly liquidity cost as estimated using each of the four proxies for a confidence level of 95% using a 60 month rolling mean, standard deviation and empirical percentile. Z is the % of zero-return days within a month. Roll is two times the square root of –1 times the autocovariance of daily returns for the month in question. FHT is the inverse of the cumulative normal distribution function of Z+1 divided by 2 which is then multiplied by 2 times the standard deviation of returns for that month. HLS is the equally weighted average of the high-low spread estimates across all overlapping two-day periods within the month.

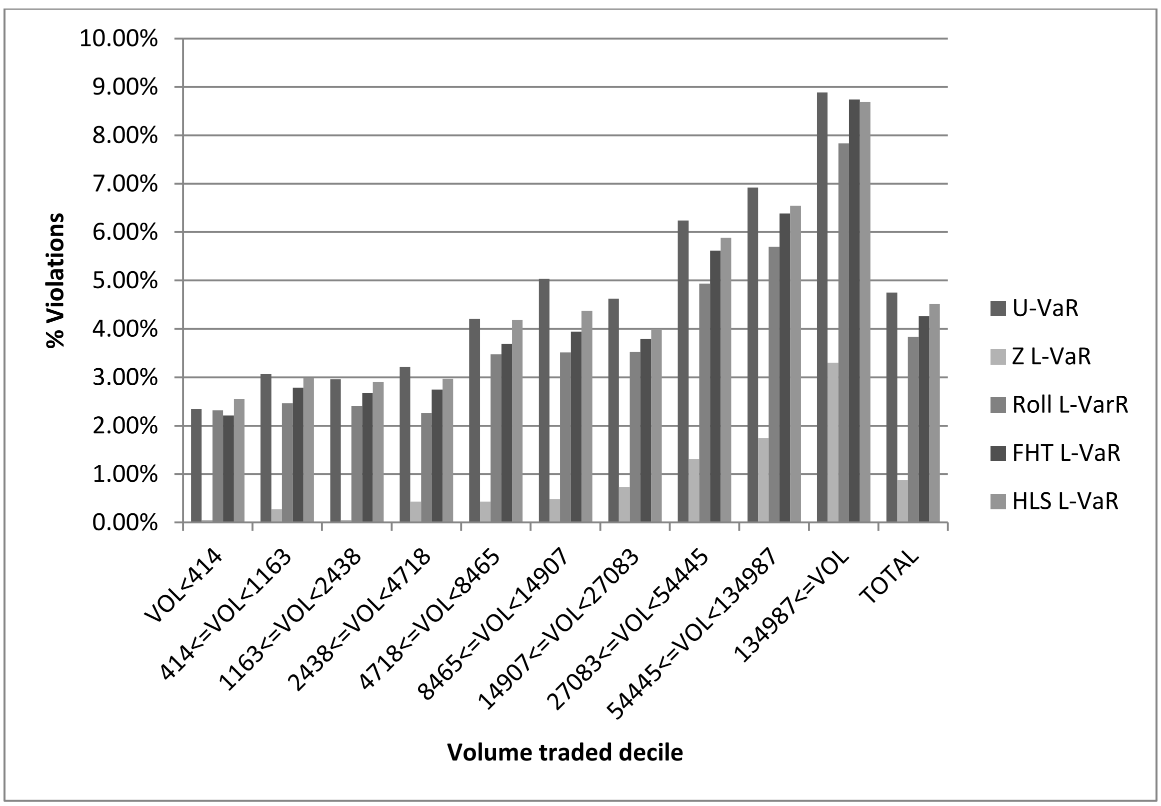

Figure 5. % Violations for VaR model sampled by volume traded using the square root of time methodology

Each model is backtested for various VaR models for each volume traded (VOL) decile. The sample includes all companies from the NYSE, Amex, and NASDAQ that were randomly selected by Hasbrouck (2006), have at least 60 months of data in CRSP, and have TAQ-based, trade-weighted proportional (%) effective spreads provided by Hasbrouck (2006) as series effcoslog. The effective spreads are averaged across days within a month. % of violations (number of violations/total observations) are reported for the monthly unadjusted VaR (U-VaR) model against unadjusted returns and the liquidity-adjusted VaR (Z L-VaR, Roll L-VaR, and FHT L-VaR) models against liquidity cost adjusted returns for each year. Liquidity-adjusted returns are calculated as log returns minus log effective cost (effcostlog). VaR are the expected monthly loss for a confidence level of 95% where the monthly return variance is by multiplying the forecasted one-day ahead standard deviation by the square root of the number of days in a month (22) from a GARCH(1,1) model. L-VaR are VaR models adjusted for liquidity costs by adding the expected average monthly liquidity cost as estimated using each of the four proxies for a confidence level of 95% using a 60 month rolling mean, standard deviation and empirical percentile. Z is the % of zero-return days within a month. Roll is two times the square root of –1 times the autocovariance of daily returns for the month in question. FHT is the inverse of the cumulative normal distribution function of Z+1 divided by 2 which is then multiplied by 2 times the standard deviation of returns for that month. HLS is the equally weighted average of the high-low spread estimates across all overlapping two-day periods within the month.

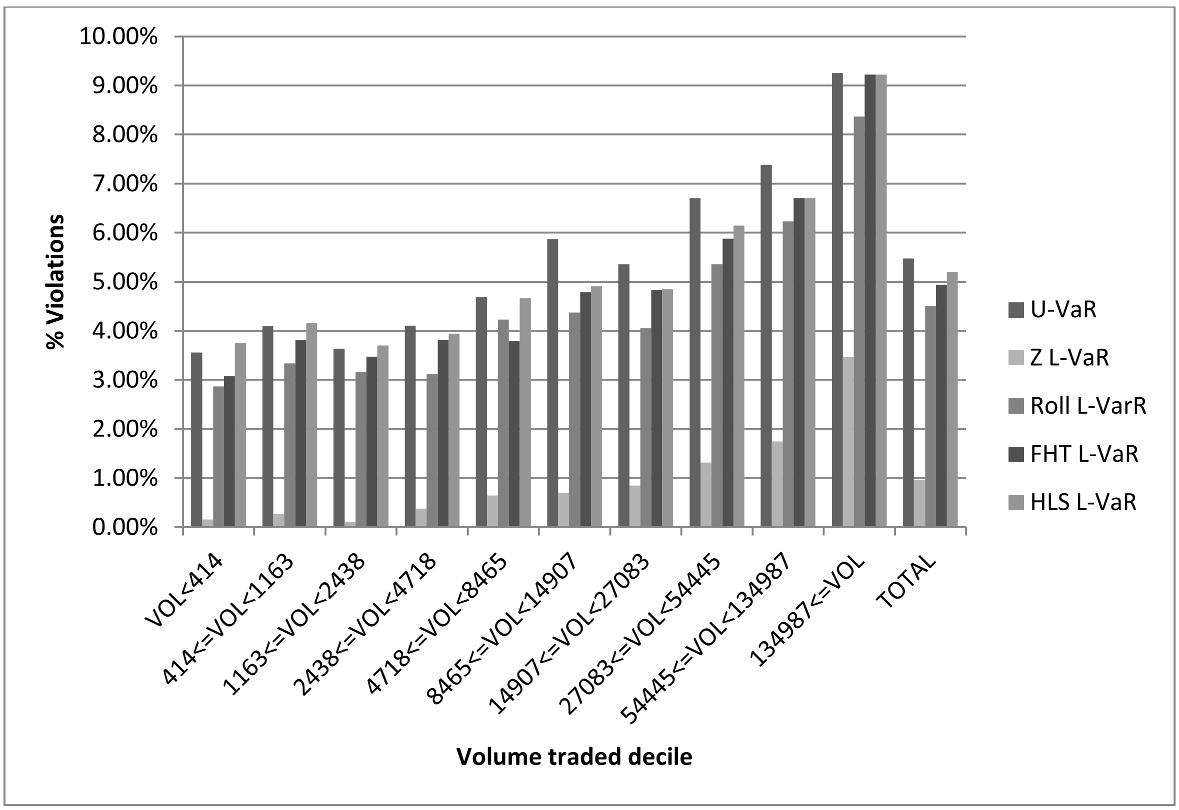

Figure 6. % Violations for liquidity adjustment sampled by volume traded

Each model is backtested for various VaR models for each volume traded (VOL) decile. The sample includes all companies from the NYSE, Amex, and NASDAQ that were randomly selected by Hasbrouck (2006), have at least 60 months of data in CRSP, and have TAQ-based, trade-weighted proportional (%) effective spreads provided by Hasbrouck (2006) as series effcoslog. The effective spreads are averaged across days within a month. Results show cross sectional % of violations (number of violations/total observations) for the monthly worst expected liquidity cost as estimated from liquidity proxies against true effective cost (effcostlog). The worst expected liquidity cost are the expected monthly liquidity cost (as estimated by the Roll, HLS, Z and FHT proxies) for a confidence level of 95% using a 60-month rolling mean, standard deviation and empirical percentile. Z is the % of 0 return days within a month. Roll is two times the square root of –1 times the autocovariance of daily returns for the month in question. FHT is the inverse of the cumulative normal distribution function of Z+1 divided by 2 which is then multiplied by 2 times the standard deviation of returns for that month. HLS is the equally weighted average of the high–low spread estimator across all overlapping 2-day periods within the month.